18 Indeed, like apparently useless arcana of nineteenth century non-Euclidean geometry, the formalism and results of general equilibrium theory are turning out to have applications undreamed of by the economists who have proved the impressive theorems in this system. For their results are being taken over and reinterpreted by mathematical ecology: stripped of their intentional interpretation, they provide proofs and stability conditions for unique stable equilibria, that modern evolutionary biology requires in the development of its own extremal theory of balance and competition in the evolution of the biosphere. Cf., for instance, Oster and Wilson, *The Social Insects* (Cambridge, MA: Harvard University Press, 1978), and R. May, *Stability and Complexity in Model Eco-Systems* (Princeton, N.J.: Princeton University Press, 1973).

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MARXIAN EXPLOITATION THEORY: A BRIEF EXPOSITION, ANALYSIS, AND CRITIQUE

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INTRODUCTION AND SUMMARY

The modern mathematical treatment of Marxian exploitation theory was developed primarily by Morishima and Seton (1961), Okishio (1963), Morishima (1973), and Wolfstetter (1973). The theory culminates in the *Fundamental Marxian Theorem*: the rate of exploitation is positive if and only if the rate of interest (= "rate of profit") is positive. According to Morishima, this theorem "may be considered as the heart and soul of Marxian philosophy..." (1973, p. 6).

The full-blown model, as in Morishima (1973), uses input-output theory and matrix algebra. However, the important results of the exploitation theory, such as the Fundamental Marxian Theorem (hereafter FMT), can be derived in an elementary one-commodity model without resorting to any matrix algebra. If the theory is successful in the one-commodity case, then it would be important to proceed to examine the n-commodity case. But if the theory is unsatisfactory even in the one-commodity case (as will be argued below) and if the problems cannot be remedied by introducing more commodities, then the n-commodity case is less important. Hence we will restrict consideration to the one-commodity model which requires no mathematical knowledge beyond some "high school algebra."

We shall argue that the flaws in the theory are at the structural and conceptual level; it attempts to use a value theory to analyze and criticize a property system. It matters not whether the value theory uses a one-commodity model or an n-commodity model; it is the wrong type of theory in the first place. The analysis of the capitalist property system should be based on a property theory, the labor theory of property (Ellerman 1980a and 1981), not a value theory such as the labor theory of value (or the neo-Ricardian or neo-classical value theories).

In particular, we shall argue that Marxian exploitation theory is in essence a reformulation of the old theory, dating from Aristotle and Aquinas, which holds that the charging of interest is exploitative. Marxian economists interpret the Fundamental Marxian Theorem (exploitation is positive if and only if interest is positive) as "revealing the inner nature" of charging interest as exploitation. In fact, the Theorem backfires and reveals that the inner nature of Marxian "exploitation" is just the charging of interest.

Marxian exploitation theory is the old Aristotelian and Scholastic "Money is barren" interest grumble dressed up in the garb of Marxian jargon. Being formulated as a value theory, it is not a critique of the institution of capitalist production. It is a critique of "exploitative" wage rates (those corresponding to positive interest rates), not a critique of the institution of wage labor itself. Accordingly, one can (squarely in the Scholastic tradition) define the "Marxian just wage," the "just price" of the commodity labor, as the wage that corresponds to a zero interest rate. Thus one can have the property relations of capitalist production and wage labor, but no Marxian exploitation if labor is paid the Marxian just wage.

Marx sent a value theory to attack a property system. It didn't even get past the guard at the door, neo-classical value theory. It takes a property theory to analyze (Ellerman 1980b and 1982) and criticize (Ellerman 1980a and 1981) a property system. Marx developed the labor theory of value, not the labor theory of property. Hence the failure of Marxian exploitation theory to develop a serious theoretical critique of capitalist production is not surprising.

POPULAR MISINTERPRETATIONS OF MARXIAN EXPLOITATION THEORY

Before turning to the Marxian model, we must comment on some prevalent misinterpretations of the model. It is a competitive model. Hence neither employers nor employees have bargaining power in the model. Yet there is Marxian exploitation in the model at positive interest rates. Hence the notion of exploitation captured by the model is not based on bargaining power. Marx wrote at length about the inequality of bargaining power in the labor market, but he wanted to critique capitalist production itself, not just the monopolistic elements in the system. Hence the model for Marxian

exploitation theory is purposely couched in the setting of competitive capitalism, not monopoly capitalism.

Some writers seem to confound Marx's writings about monopoly capitalism with the exploitation theory in its (hypothetical) competitive setting.

> Thus, in Marx's view, the weak bargaining position of labor and the pressure of unemployment both operate systematically to maintain the value of the commodity labor power below the level to which it would otherwise be forced by the competitive bidding capitalists. (Heilbroner 1980, p. 110)

The Marxian model presented below is fully competitive (no inequality of bargaining power and no reserve army of the unemployed), but there is still Marxian exploitation. Hence it is not a bargaining power theory of exploitation. There is already a neo-classical or "bourgeois" theory of exploitation based on monopolistic elements which cause the workers to receive less than the full value of their marginal product (e.g., Pigou 1962, Chapter XIV; Robinson 1948, Book IX). Many Marxists have retreated from the attempt to critique the model of competitive capitalism and have similarly focused their critical analysis on the monopolistic elements in monopoly capitalism.

An even more popular misinterpretation of Marxian exploitation theory is based not on market power but on the power relations in the labor process itself. Following Marx himself, Braverman (1974), Marglin (1974), and many other writers have emphasized how the employers can use their power inside the workplace to structure and control the worker's activity to maximize the employer's advantage. This analysis is usually couched in the jargon of Marxian exploitation theory (Marglin being an exception to this rule). The bosses are described as using their workplace power to force the workers to work longer and harder than is necessary to reproduce the value of their labor power—and so forth. But this is a complete misunderstanding of Marxian exploitation theory.

Workplace power relations play no role whatsoever in the model of Marxian exploitation theory. This is almost incomprehensible to Marxists unfamiliar with the modern mathematical formulation of the theory. It is one of the reasons why the precise mathematical formulation of the model has been so important. It allows one to separate the theory itself from the veneer of jargon which almost invariably accompanies the theory in its literary presentations. There are several ways to see the irrelevance of workplace power relations to the FMT. It is simplest just to observe that the model never needs or uses any assumptions about the employer's control over the employees. The FMT does not even depend on, much less "reveal the inner workings of," workplace power relations.

This irrelevance thesis can also be proven by constructing explicit countermodels. One can construct a model of a worker self-managed market economy with a positive interest rate. Each firm is democratically self-managed by the workers who work in it, so the traditional power relations of the wage labor contract are absent. Yet so long as the workers pay interest on borrowed capital, the expected Aristotle-Aquinas-Marx conclusion that the workers are exploited can be derived using Marxian exploitation theory. Wolfstetter sees this conclusion so he wisely decides to simply not apply the Marxian exploitation concept to the worker-managed case (1973, p. 799). Even the most flawed theory can be spared embarrassment by choosing not to apply the theory to the cases in which it doesn't work.

It is also possible to construct countermodels where there is an employment relation and Marxian exploitation, but where it is the employees who exploit the employer (Ellerman 1977). These models are, to be sure, rather unconventional. They are non-Marxian models of Marxian exploitation theory in a manner analogous to the non-Euclidean models of geometry. The non-Euclidean models show the independence of the parallel postulate from the other axioms of Euclidean geometry. The non-Marxian models logically prove that the FMT is independent of workplace power relations since the exploitation relation cuts against the power relations in the models.

The rationale for the non-Marxian models is simple. The FMT reflects in Marxist terms the old interest grumble that lenders or creditors gain and borrowers or debtors lose when the interest rate is positive. In the conventional Marxian model, the employer has a role financially equivalent to a lender's role so the employer gains (i.e., the employer exploits the employees) when the interest rate is positive. A non-Marxian model can be constructed where the employer has a borrower's role instead of a lender's role, and where the equilibrium interest rate is positive. The workplace power analysis would predict as usual that the employer exploits the employees, but Marxian exploitation theory now grinds out exactly the opposite result. The employees exploit the employer. The FMT only reflects the financial role of the employer vis-à-vis the interest rate. When the employer has a borrower's role with a positive interest rate, then in accordance with the interest grumble analysis the employer is the exploited one (since borrowers lose under a positive interest rate). But these non-Marxian models would amount to overkill in the context of this brief analysis and critique of Marxian exploitation theory. The irrelevance of workplace power relations to the FMT can be seen by clearly understanding the standard model of Marxian exploitation theory presented below.

THE ONE COMMODITY MODEL: THE QUANTITY SYSTEM

In the one-commodity competitive model (e.g., von Weizsacker [1971]), there is one produced good, such as corn, and homogeneous labor. There are no non-produced goods such as land or natural resources. There is no fixed capital, no joint products and no uncertainty in the model. The *technical coefficient* A is the positive number of units of the commodity which must be used up in order to produce a unit of the commodity as output, e.g., the number of bushels of seed corn needed per bushel of harvest corn. For any non-zero number x, the reciprocal of x can be written as $x^{-1} = 1/x$. If x is between -1 and $+1$, i.e., $-1 < x < +1$, then from high school algebra we know that the infinite geometric series $1 + x + x^2 + \ldots$ has a sum, and that the sum is $[1 - x]^{-1} = 1/(1 - x)$, i.e.,

$$1 + x + x^2 + \ldots = 1/(1 - x) = [1 - x]^{-1}$$

for $-1 < x < +1$. A technical coefficient A is said to be *viable* if the infinite sum $1 + A + A^2 + \ldots$ exists, in which case it equals $[1 - A]^{-1}$, and is positive. The technical coefficient is assumed to be positive and viable so $0 < A < 1$. Viability means that less than a unit of the good is needed to produce a unit of the good.

Let a_0 be the positive *labor coefficient* which specifies that a_0 units of labor (i.e., labor time) are required to produce one unit of the output. Thus a positive gross output level x requires the commodity inputs Ax and the labor inputs a_0x. It takes one time period called a "year" for the labor a_0x to produce the outputs x by using the inputs Ax. The inputs are required at the beginning of the year and the outputs are available at the end of the year.

We will first give the basic definitions and formally develop the Marxian exploitation theory. The definitions and the theory will be analyzed later. The fundamental definition is that of the *Marxian value* or *labor content* per unit of the commodity:

$$v := a_0[1 - A]^{-1} = a_0[1 + A + A^2 + \ldots],$$

where ":=" means "equal by definition." It is assumed that the labor is hired and that the real *wage basket* is z units of the commodity received at the beginning of the period per unit labor. Thus the real wage is z. One may construe the wage baskets as being paid directly to the workers by their employers or as being purchased by the workers with their money wages. In return for each unit of labor sold, each worker receives z units of the commodity with the labor content of Marxian value vz. Hence vz is called the *necessary labor* or *paid labor*, and the remainder $1 - vz$ is called the *surplus labor, unpaid labor* or *surplus value* per unit labor. The *Marxian rate of exploitation* e is defined as the ratio of surplus or unpaid labor over the necessary or paid labor:

$$e := \text{surplus labor/necessary labor} = (1 - vz)/vz.$$

If x is the gross product, then $y := x - Ax = [1 - A]x$ is called the *net product*. The labor required to produce x is a_0x so the real wage bill is za_0x and the remaining product

$$s := x - Ax - za_0x = y - za_0x$$

is called the *surplus product*. Since a_0x units of labor are performed, each containing $1 - vz$ units of surplus labor, the total surplus labor is

$$\begin{aligned} &[1 - vz]a_0x \\ &= (a_0x - vza_0x) \\ &= (a_0[1-A]^{-1}[1-A]x - vza_0x) \\ &= v(x - Ax - za_0x) \end{aligned}$$

where the last term is vs, the labor content of the surplus product. Thus, the total surplus labor is the labor content of the surplus product.

If the workers' real wage baskets are construed as technical input requirements, like the feed for horses, then the *augmented technical coefficient* $A^* = A + za_0$ gives the total amount of the good needed, both as an input and as a wage good for the required labor, in order to produce a unit of the good. Since $evz = 1 - vz$, we may multiply by a_0 to obtain:

$$\begin{aligned} &evza_0 \\ &= a_0 - vza_0 \\ &= a_0[1-A]^{-1}[1-A] - vza_0 \\ &= v[1 - A - za_0] \\ &= v[1 - A^*]. \end{aligned}$$

Since v, z, and a_0 are all positive, vza_0 is positive. Hence if the rate of exploitation e is positive, then so is $evza_0 = v[a - A^*]$. Since v is positive, the positivity of $v[1 - A^*]$ implies that $1 - A^*$ is positive, i.e., that the augmented technical coefficient A^* is less than one. Hence positive exploitation implies the viability of the augmented technology. With gross output x, the surplus product is $s = [1 - A^*]x$ and the total surplus labor is $vs = v[1 - A^*]x$.

THE ONE COMMODITY MODEL: THE PRICE SYSTEM

We have so far developed only the *quantity system*, since only physical quantities and no money prices have been considered. The *price system* can now be developed. Let p^* be the money price per unit of the commodity, let w be the money wage rate, and let r be the rate of interest (or "rate of profit") for the time period. Since wages are paid and inputs are purchased at the beginning of each time period, the capital outlay required per unit output is $wa_0 + p^*A$. Since the interest rate (as a decimal, not a per cent) is r, the market will trade $wa_0 + p^*A$ units of capital at the beginning of the time period for $(1+r)[wa_0 + p^*A]$ units of capital at the end of the time period. That is the passive use of capital, loaning it out at interest. The "active" use of capital is to invest it in production by purchasing the inputs, and then appropriating and selling the outputs. The initial capital $wa_0 + p^*A$ purchases the inputs which produce one unit of output at the end of the period. That unit sells for p^* so the "active" use of capital converts $wa_0 + p^*A$ into p^* units of capital at the end of the period. In a perfectly competitive model with no uncertainty, capital can be freely switched between its "active" and passive uses, so arbitrage will enforce equality between the two returns in equilibrium. Hence we have the:

Competitive Arbitrage Condition: $p^* = (1+r)[wa_0 + p^*A]$.

It is assumed that the workers do not save, so the money wage w per unit labor will just purchase the real wage basket z, i.e., $w = p^*z$. Substituting for w in the above competitive equilibrium condition, we have:

$$p^* = (1+r)[p^*za_0 + p^*A] = (1+r)p^*A^*.$$

Hence for non-zero p^*, we have

$$r = (1 - A^*)/A^*$$

from $p^*[1 - (1+r)A^*] = 0$. By directly solving the equilibrium condition, we also have:

$$p^* = (1+r)wa_0[1 - (1+r)A]^{-1}.$$

Since $[1 - (1+r)A^*] = 0$ and A^* is larger than A, $[1 - (1+r)A]$ is positive and thus $p^* = (1+r)wa_0[1 - (1+r)A]^{-1}$ is positive.

If the equilibrium price p^* of the commodity is expressed in terms of labor by normalizing the money wage rate w to 1, the normalized price is:

$$
\begin{aligned}
p &:= p^*/w \\
&= (1+r)a_o[1 - (1+r)A]^{-1} \\
&= (1+r)a_o[1 + (1+r)A + (1+r)^2 A^2 + \ldots].
\end{aligned}
$$

THE FUNDAMENTAL MARXIAN THEOREM

We are finally in a position to prove the FMT.

Fundamental Marxian Theorem: The rate of exploitation e is positive if and only if the rate of interest r is positive.

Proof: If e is positive, then we proved above that the augmented technology A^* was viable, i.e., $A^* < 1$. Hence $r = (1 - A^*)/A^*$ is positive. Conversely, if r is positive, then $p > v$ since

$$p = (1+r)a_o[1 + (1+r)A + (1+r)^2 A^2 + \ldots] > a_o[1 + A + A^2 + \ldots] = v.$$

Hence, using $p^*z = w$, we have $pz = p^*z/w = 1 > vz$ so $e = (1 - vz)/vz$ is positive.

THE DEFINITION OF MARXIAN VALUE

There are three different ways to define labor content or Marxian value. Under certain conditions, all three methods yield the same formula: $v = a_o[1 - A]^{-1}$. The three methods will be referred to as: (1) the historical method, (2) the reproduction method (due to Wolfstetter), and (3) the net product method.

The *historical method* defines the labor content of a unit of a commodity as the labor historically embodied in the commodity (under certain technical conditions). If the technical coefficients A and a_o are assumed to have been constant, then one can work backwards to "dissolve" the commodity into labor. One unit of the output requires the commodity inputs A and the labor input a_o. The inputs A require the inputs A^2 and the labor a_oA. The inputs A^2 require the inputs A^3 and the labor a_oA^2, and so forth. This "historical" breakdown is summarized in the following table.

PERIOD	1	0	−1	−2	...	−n	...
COMMODITY	1	A	A^2	A^3	...	A^{n+1}	...
LABOR		a_o	a_oA	a_oA^2	...	a_oA^n	...

TABLE 1: Reduction to Past Embodied Labor

The sum of the LABOR row gives the historically embodied labor:

$$a_o + a_oA + a_oA^2 + \ldots = a_o[1 + A + A^2 + \ldots] = a_o[1 - A]^{-1} = v.$$

The above computation assumes that the technical coefficient was always A. A more realistic assumption would allow technical change with a technical coefficient A_n n periods in the past. Then the sequence of commodity inputs would be A_1, A_1A_2, $A_1A_2A_3$, and so forth. Still assuming a constant labor coefficient, the required labor inputs are a_o, a_oA_1, $a_oA_1A_2$, and so forth. Without knowing all the A_i's, one cannot know the value of the historically embodied labor

$$a_o + a_oA_1 + a_oA_1A_2 \ldots .$$

Thus the "historical" method is best interpreted as defining the "socially necessary" labor required to produce a unit of the commodity if the best available technology, i.e., the present technology, is always used.

The *reproduction method* adds up the present labor required to produce a steady stream of one unit of the commodity in each future period—instead of adding up the past labor required to produce one unit of the commodity in the present. One unit of the commodity one period from now requires a_o units of present labor. Another unit of the commodity two periods from now requires commodity inputs A one period from now which, in turn, requires a_oA units of present labor. One unit of output three periods from now requires A inputs two periods hence which, in turn, requires A^2 inputs one period hence which, in turn, requires a_oA^2 units of present labor—and so forth. The present labor requirements for the steady reproduction of one unit of output are indicated in the following table.

PERIOD	0	+1	+2	+3	...
COMMODITY	A	1			
LABOR	a_o				
COMMODITY	A^2	A	1		
LABOR	a_oA	a_o			
COMMODITY	A^3	A^2	A	1	
LABOR	a_oA^2	a_oA	a_o		
.	.	.	.	.	
.	.	.	.	.	
.	.	.	.	.	

TABLE 2: Steady Reproduction Labor Requirements

The present (period 0) labor required for the steady reproduction of a unit of the commodity is:

$$a_o + a_oA + a_oA^2 + \ldots = a_o[1 - A]^{-1} = v.$$

Once again, it is unrealistic to literally interpret this calculation since it assumes an unchanging technology throughout the future. Hence it is best construed as the calculation of the present labor required to steadily reproduce a unit of the commodity if the present technology is always used.

The *net product method* avoids the infinite past and the infinite future by defining the labor embodied in one unit of a commodity as the present labor required to produce the *net* product of one unit of the commodity. If x is the gross product, the net product is $y = x - Ax = [1 - A]x$. Hence if the net product is $1 = y = [1 - A]x$, then the gross product is $x = [1 - A]^{-1}$ and the required labor is $a_ox = a_o[1 - A]^{-1} = v$.

ANALYSIS OF THE MARXIAN VALUE DEFINITION

The basic definition of Marxian value systematically neglects the effect of time, an effect which is economically registered by the rate of interest. Time puts a difference on commodities. Units of a commodity or of labor at different points in time are economically distinct—like "apples and oranges." One unit of a commodity or labor at time t is *economically equivalent* to $1+r$ units at time $t+1$ in the sense that one unit at time t would trade on the market for $1+r$ units at time $t+1$. For example, by loaning money out at interest, one dollar at time t is exchanged for $1+r$ dollars at time $t+1$. Each of the definitions of Marxian value treats units of the commodity or labor at different points in time as being the same. Hence the definition of Marxian value implicitly sets the interest rate to zero, and the analysis of exploitation is based on that zero-interest Marxian value definition. Thus it is hardly surprising that one can derive that there is "exploitation" according to those definitions if and only if the interest rate is larger than zero, i.e., the Fundamental Marxian Theorem.

Consider the net product definition of Marxian value. The very notion of the net product $y = x - Ax$ assumes that the beginning-of-the-year inputs Ax are commensurate with the end-of-the-year product x so that the former can be subtracted from the latter to arrive at the net product. However, since time makes commodities distinct, the difference $x - Ax$ is as meaningful as the difference "5 apples minus 3 oranges." The inputs Ax are equivalent to $(1+r)Ax$ units at the end of the year, so the time-corrected net product in terms of commodities timed with outputs is

$$y(r) := x - (1+r)Ax.$$

Hence for a corrected net product of $1 = y(r)$, the gross output is

$$x = [1 - (1+r)A]^{-1},$$

so the time-corrected net product definition of Marxian value is

$$v(r) := a_o[1 - (1+r)A]^{-1},$$

in terms of beginning-of-the-year labor.

The neglect of time in the historical definition of Marxian value occurs in the addition of the labor performed in different time periods. If one unit of the commodity is produced at year's end, then all the past embodied labor can be transformed into the equivalent beginning-of-the-year labor before being summed. The labor a_oA^n performed n years before the beginning of the current year is equivalent to $a_o(1+r)^n A^n$ units of labor at the beginning of the year. Hence the corrected historical definition of Marxian value is:

$$\begin{aligned} & a_o + a_o(1+r)A + a_{(1+r)}^2 A^2 + \ldots \\ &= a_o[1 + (1+r)A + (1+r)^2 A^2 + \ldots] \\ &= a_o[1 - (1+r)A]^{-1} \\ &= v(r) \end{aligned}$$

(assuming that $(1+r)A < 1$) which is the same as the corrected net product definition.

The reproduction definition neglects time by considering the physically constant stream of one unit of the commodity in each future time period as constituting "steady reproduction." With a positive time discount, a physically constant stream is economically declining. The stream that is economically steady, in terms of end-of-the-year output, is:

$$1, (1+r), (1+r)^2, \ldots,$$

and the beginning-of-the-year labor necessary to produce that stream is

$$a_o + a_oA(1+r) + a_oA^2(1+r)^2 + \ldots = v(r).$$

The time-corrected Marxian value v(r), obtained by all three methods, gives the total labor, measured in terms of beginning-of-the-year labor, embodied in a unit of end-of-the-year output. The Marxian value in terms of labor timed together with output (instead of labor lagged one period back) is

$$(1+r)v(r) = (1+r)a_0[1 - (1+r)A]^{-1} = p(r),$$

which is the *price* of output in terms of labor, as a function of r. Hence the difference between Marxian value and competitive market price vanishes after the time correction. It should also be noted that the competitive market price p(0) corresponding to an interest rate of zero is just the original "uncorrected" Marxian value $v = v(0) = p(0)$.

Since each wage basket z was received at the beginning of the year, the equivalent real wage in terms of end-of-the-year output is $(1+r)z$, and its beginning-of-the-year embodied labor is the time-corrected necessary labor $v(r)(1+r)z$ per unit labor. Hence the time-corrected surplus labor is $1 - v(r)(1+r)z$. But $(1+r)v(r) = p$ and $pz = 1$, so the time-corrected surplus labor is identically equal to zero. Thus the rate of exploitation, being the ratio of surplus labor over necessary labor, is also identically equal to zero. By ignoring the effect of time, the original Marxian value definition yields the Fundamental Marxian Theorem that there is exploitation if and only if the interest rate is positive. By inserting the effect of time, as measured by the interest rate, into the analysis, the notion of "exploitation" disappears.

ANALYSIS OF THE FMT AS AN INTEREST GRUMBLE

We shall argue that the Fundamental Marxian Theorem (FMT) is essentially a translation into Marxian jargon of the old:

Interest Grumble: Creditors gain if and only if the rate of interest r is positive.

Consider a party at the beginning of the year with the financial capital $[a_0 + pA]x$ for some positive x. This party has two options, to use the capital passively or "actively." The party could have the passive creditor role during the year by loaning the capital out at interest and then receiving back $(1+r)[a_0 + pA]x$ at the end of the year. Alternatively, the party could take on the "active" employer role during the time period by purchasing the inputs Ax and the labor a_0x and by appropriating and selling the output x for px at the end of the year. Hence the party could transform the capital $[a_0 + pA]x$ into either $(1+r)[a_0 + pA]x$ or px at the end of the year. Competitive arbitrage insures that the creditor's and the employer's role are

financially equivalent in the sense of having equal returns. That arbitrage-enforced equivalence between the two roles is the basis for the competitive equilibrium condition:

$$px = (1+r)[a_0 + pA]x.$$

If the time-correction were applied to the surplus product, then the corrected surplus product would be:

$$s(r) = y(r) - (1+r)za_0x = [1 - (1+r)(A + za_0)]x = [1 - (1+r)A^*]x.$$

Using $pz = 1$ in the competitive equilibrium condition, we have:

$$px = (1+r)[pza_0 + pA]x = (1+r)p[za_0 + A]x = (1+r)pA^*x,$$

so the market value of the corrected surplus product is:

$$ps(r) = p[1 - (1+r)A^*]x = 0.$$

If we transfer the interest cost rpA^*x to the other side of the equation, we have:

$$\text{market value of surplus product} = ps = p[1 - A^*]x = rpA^*x = \text{interest cost.}$$

Hence the "uncorrected" surplus product $s = [1 - A^*]x$ is just the interest cost expressed in terms of output.

The gain to the creditor in the passive use of the capital is that interest payment rpA^*x. The Marxian surplus product notion provides a way to translate the creditor's gain into *production language* since the active use of the capital is to undertake production. The surplus product s is the amount of output that is appropriated and sold to cover the interest costs (costs which are explicit if the capital were borrowed and implicit if the employer supplied the capital). Hence the interest grumble, creditors gain if and only if (iff) r is positive, translates into production language as, employers appropriate a positively valued surplus product iff r is positive.

The major concept in Marxian exploitation theory is the notion of Marxian value v. We have already noted that v is just the competitive equilibrium price when the interest rate is zero, i.e., $v = p(0)$. Hence the Marxian value calculations really amount to comparing the actual model (where we may assume that r is positive) to the *benchmark regime* where the interest rate is stipulated to equal zero. In a multi-commodity model, the wage basket z could be expanded proportionately to a wage basket z* so that the corresponding equilibrium interest rate would be zero. In our simple one commodity model, we can exploit the $p(r)z(r) = 1$ relationship to solve for the wage basket z(r) as a function of r:

$$z(r) = 1/p(r).$$

Hence in the benchmark regime where $r = 0$,

$$z^* := z(0) = 1/p(0) = 1/v = [1 - A]/a_0 = \text{net product per unit labor.}$$

This z* is the *Marxian just wage* (in terms of the commodity) since the corresponding rate of exploitation is $e = (1 - vz^*)/vz^* = 0$. According to Morishima, it "represents the real wage rate that would prevail if there was no exploitation" (1973, p. 54).

In the actual model, where we may assume that the interest rate r is positive, the price of output is $p = p(r)$ and the wage basket is $z = z(r)$. In the benchmark regime, the interest rate is stipulated to equal zero, the price of output is the Marxian value $v = p(0)$ and the wage basket is the Marxian just wage $z^* = z(0)$. Labor is the numeraire in both cases, so one unit of labor has the market value of $1 = p(0)z(0) = p(r)z(r)$. As r varies, p(r) and z(r) vary so that their product has the constant value of 1, so they would graph as a rectangular hyperbola.

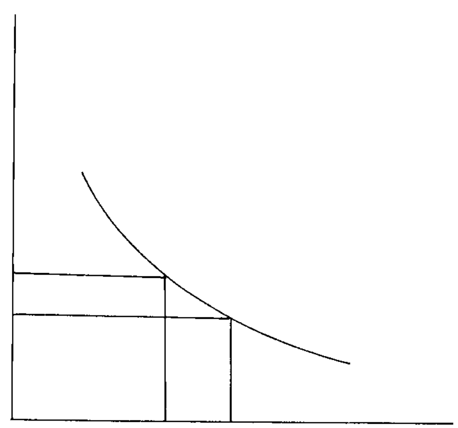

Diagram 1: The Price-Wage Tradeoff

The areas of the two large overlapping rectangles are equal to unity, i.e., $p(0)z(0) = p(r)z(r) = 1$. The overlap has the area $p(0)z(r) = vz$. It is the real wage in the actual model evaluated at the zero-interest benchmark prices, and it is called the "necessary labor" per unit labor. Since the area of the two big (equal) rectangles represents the price of labor (unity) in each model, subtracting the overlap from each rectangle leaves two equal representations (the top border and right border rectangles) of the remaining value of a unit of labor—which is called the "surplus labor" per unit labor. The equal areas of the two border rectangles give two expressions for the surplus labor:

$$[p(r) - p(0)]z(r) = p(0)[z(0) - z(r)].$$

The left-hand side is

$$[p(r) - p(0)]z(r) = p(r)z(r) - p(0)z(r) = 1 - vz,$$

the surplus labor per unit labor, and the right-hand side is

$$p(0)[z(0) - z(r)] = v[z^* - z] = v[(1-A)/a_0 - z] = v[1-A-za_0]/a_0,$$

which is the Marxian value of the surplus product per unit labor. Multiplying each side by the total labor a_0x yields the previous result that the total surplus labor, $(1 - vz)a_0x$, equals the Marxian value of the surplus product, $v[1-A-za_0]x = vs$.

MARXIAN EXPLOITATION THEORY "GENERALIZED"

The Marxian exploitation analysis is illuminated by "generalizing" it to apply to any drop in the price of a commodity sold. Let X and Y be commodities where a unit of Y is sold for a certain quantity of X (X could be corn or any other commodity). Let Y be the numeraire and let P_0 be the initial price of a unit of X in terms of Y. One unit of Y is then sold for $X_0 = 1/P_0$ units of X. Now suppose that the price of Y drops relative to other commodities. Since Y is the numeraire, this means an increase in the price of X to P_1. Then one unit of Y sells for the smaller amount, $X_1 = 1/P_1$ units of X. This is illustrated in Diagram 2 where the slope of the lines is minus the price of X.

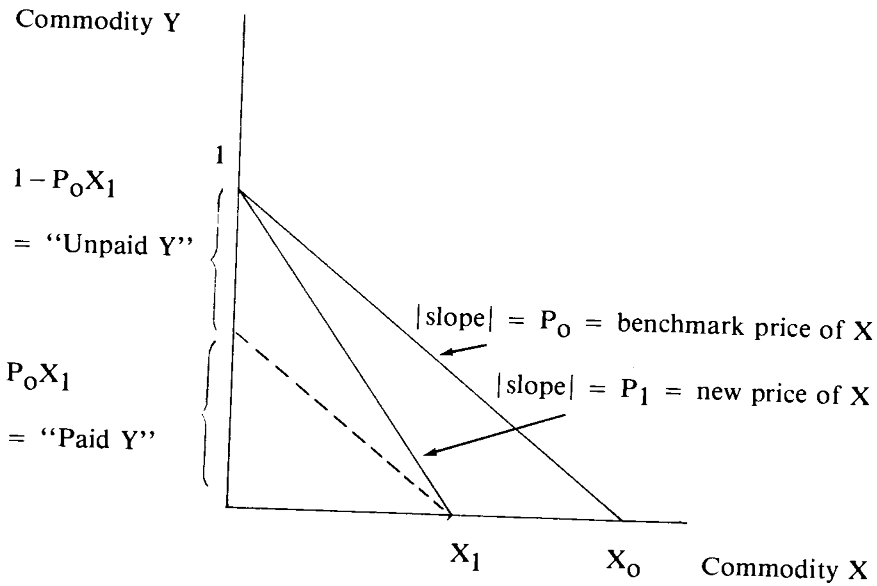

Diagram 2: "Exploitation" Analysis of Any Price Change

There were two price regimes, the original "benchmark" price of P_o and the actual ex post price P_1. If we think of the ex post transaction in terms of the ex ante or benchmark price, then the one unit of Y is sold for X_1 which has the "value" of P_oX_1. Hence that is the portion of the unit of Y that is "really" being paid for in benchmark terms; it is the "paid Y" per unit Y. The remainder, $1 - P_oX_1$, is the "unpaid" or "surplus" Y per unit Y.

As before, the surplus Y can be expressed as the result of either a price effect or a quantity effect:

$$[P_1 - P_o]X_1 = P_o[X_o - X_1].$$

We have "pierced the veil" of the actual (ex post) market transaction to reveal its "inner nature." In the sale of a unit of Y, the seller first gives up the portion P_oX_1 and is paid in return X_1 units of X which has the same "value." Everything seems fair and square. But then the seller is "forced to alienate" the remaining $1 - P_oX_1$ which is "appropriated as surplus Y" by the buyer without any further payment in return. In this manner, one can "strip away the outer disguise" of a price change to "lay bare its innermost

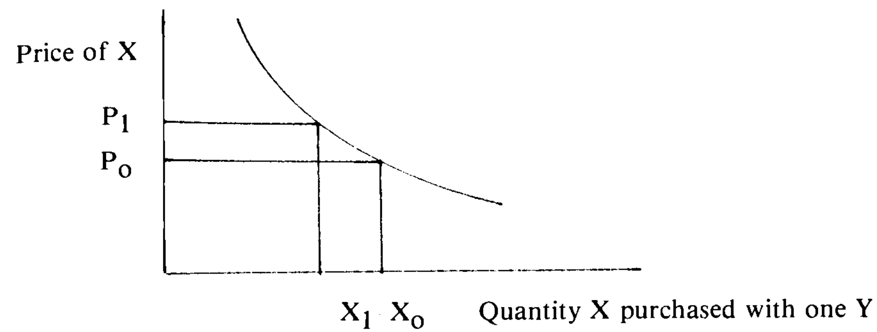

Diagram 3: The Price-Quality Purchased Tradeoff

workings." "Beneath the facade" of the free market transaction, we see the "exploitation" of the Y seller by the "forced alienation of surplus Y." The rate of exploitation e would be defined as the ratio of the unpaid Y over the paid Y:

$$e := (\text{Unpaid Y})/(\text{Paid Y}) = (1 - P_oX_1)/P_oX_1).$$

Then the "heart and soul" of this philosophy could be expressed by the following generalized price grumble:

"Fundamental Exploitation Theorem": $e > 0$ if and only if $P_1 > P_o$.

In the transition from the perhaps hypothetical benchmark regime to the actual regime, the price of Y in terms of other commodities has dropped, so the price of a commodity X in terms of Y increases from P_o to P_1. The "exploitation" reflects the profound fact that those who sell Y are worse off given a drop in the price of Y in terms of other commodities (where "worse off" and "drop" are relative to the benchmark regime).

In Marx's use of this methodology, time is not economically relevant in the benchmark model. In the hypothetical transition from the zero-interest benchmark model to the actual model, time enters as a scarce resource commanding a positive price (the positive interest rate). The price p of the commodity in terms of labor is a direct monotonic function of the interest rate r:

$$p(r) = (1+r)a_o[1-(1+r)A]^{-1} = (1+r)a_o[1 + (1+r)A + (1+r)^2 A^2 + \ldots].$$

Hence "$p = p(r) > p(0) = v$" if and only if "$r > 0$." Thus, taking "$p > v$" as the price increase "$P_1 > P_0$" in the above theorem, we can substitute the equivalent condition "$r > 0$" and obtain, as a corollary, the

Fundamental Marxian Theorem: $e > 0$ if and only if $r > 0$.

That is the whole sum and substance of Marxian exploitation theory. It is just the above analysis of a "price change" where the hypothetical ex ante situation is the benchmark regime of a zero interest rate. When stripped of the "outer disguise" of Marxian jargon, the exploitation analysis can be applied to any price change and has nothing whatsoever to do with the property relations or power relations inside or outside of the workplace. The force of the Marxian "exploitation" analysis amounts to postulating that the benchmark prices corresponding to zero interest are "just" or "true" prices. By viewing transactions made at actual prices in terms of these "just" prices, one can reveal the innermost workings of Marxian exploitation theory. There is an old and venerable tradition of thought, prominently including Aristotle and Aquinas, which holds that charging interest is exploitation. Marxian exploitation theory turns out to be the greatest in that long line of interest grumbles.

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