Spencer-Brown and the logic of partitions on a two-element set

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The idea is to interpret his original interpretation about distinctions as just reasoning in the partition algebra of the two element set, say $\{a, b\} = 2$ where $0 = \{\{a, b\}\}$, the blob and $1 = \{\{a\}, \{b\}\}$, the discrete partition. Then his operation $\neg$ is short for $\square$ which distinguishes the inside from the outside and would be represented as moving from the blob $0 = \{\{a, b\}\}$ to $1 = \{\{a\}, \{b\}\}$. As an operation, it is just applying 1. Then the law of calling $\neg\neg = \neg$ would be interpreted as applying the discrete partition twice, i.e., $1 \vee 1 = 1$. The other operation, represented as exponentiation $\neg^{\neg}$ has the truth table of complementation $1 \Rightarrow 0$ so $1 \Rightarrow 0 = 0$ and $(1 \Rightarrow 0) \Rightarrow 0 = 1$ although perhaps it is $1 = \neg$ and $\neg^{\neg} = 1 \Rightarrow 0 = 0$.

The primary algebra then is the partition algebra $\prod(2)$ which is isomorphic to $\wp(1)$, the powerset BA on the one-element set. But when unknowns are used and laws are derived, then they may not be true in the partition algebras of larger sets. One is that the negation of the law of excluded middle is a contradiction, i.e., equals the blob:

$[(p \Rightarrow 0) \vee p] \Rightarrow 0 = 0$ which is true in any partition algebra. He calls that the law of position. But another formula is the law of transposition:

$$[((p \vee r) \Rightarrow 0) \vee ((q \vee r) \Rightarrow 0)] \Rightarrow 0 = [[(p \Rightarrow 0) \vee (q \Rightarrow 0)] \Rightarrow 0] \vee r.$$

Is this true in general? Take $p = r \neq 0$ and $q \neq 0$ so the LHS is $(p \Rightarrow 0) \Rightarrow 0$ and the RHS is p which does not hold for $p \neq 1, 0$.