Determination through Universals: An Application of Category Theory in the Life Sciences

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Abstract

Category theory has foundational importance because it provides conceptual lenses to characterize what is important and universal in mathematics—with adjunctions being the primary lense. If adjunctions are so important in mathematics, then perhaps they will isolate concepts of some importance in the empirical sciences. But the applications of adjunctions have been hampered by an overly restrictive formulation that avoids heteromorphisms or hets. By reformulating an adjunction using hets, it is split into two parts, a left and a right semiadjunction (or half-adjunuction). Semiadjunctions (essentially a formulation of a universal mapping property using hets) turn out to be the appropriate concept for applications in the life sciences. The semiadjunctions characterize three principal schemes with applications:

1. determination through a receiving universal (e.g., natural selection, perception, language acquisition, recursion, and language understanding);

2. determination through a sending universal (e.g., intentional action, DNA mechanism to build amino acids, hierarchy of regulatory genes to build organs, stem cells, and language production), and

3. two-way determination with one universal as both receiving and sending (e.g., perception/action and language understanding/production).

Contents

1 Introduction

Category theory has foundational importance because it provides conceptual lenses to characterize what is important and universal in mathematics—with an adjunction (or pair of adjoint functors) being the primary lense. The mathematical importance of adjunctions is now well recognized. As Steven Awodey put it in his recent text:

> The notion of adjoint functor applies everything that we have learned up to now to unify and subsume all the different universal mapping properties that we have encountered, from free groups to limits to exponentials. But more importantly, it also captures an important mathematical phenomenon that is invisible without the lens of category theory. Indeed, I will make the admittedly provocative claim that adjointness is a concept of fundamental logical and mathematical importance that is not captured elsewhere in mathematics. $[1, p.179]$

Other category theorists have given similar testimonials.

> To some, including this writer, adjunction is the most important concept in category theory. [26, p. 6]

> The isolation and explication of the notion of *adjointness* is perhaps the most profound contribution that category theory has made to the history of general mathematical ideas.[13, p. 438]

> Nowadays, every user of category theory agrees that [adjunction] is the concept which justifies the fundamental position of the subject in mathematics. [25, p. 367]

If a concept, like that of a pair of adjoint functors, is of such importance in mathematics, then one might expect it to have applications, perhaps of some importance, in the empirical sciences. Yet this does not seem to be the case, particularly in the life sciences. Perhaps the problem has been finding the right level of generality or specificity where non-trivial applications can be found, i.e., finding out "where theory lives."

This paper argues that the application of adjoints has been hampered by an overly specific formulation of the adjunctive properties that only uses homomorphisms or homs[1] (object-to-object morphisms *within* a category). A reformulation of adjunctions using heteromorphisms or hets (object-to-object morphisms between objects of *different* categories) allows an adjunction to be split into two "semiadjunctions." The argument is that a semiadjunction (essentially a reformulation of a universal mapping property using hets) turns out to be the right concept for applications. Moreover, by "splitting the atom" of an adjunction into two semiadjunctions, the semiadjunctions can be recombined in a different way to define the cognate notion of a "brain functor"–which, as the name indicates, has applications in cognitive science.

This application of category theory in the life sciences is not exact in the sense that, say, calculus is applied in physics. The category theory is exact at the mathematical level, but it is the general determinative schema of the semiadjunctions that is applied.

[1] "Hom" is pronounced to rhyme with "Tom" or "bomb."

There is already a considerable but widely varying literature on the application of category theory to the life sciences–such as the work of Robert Rosen [24] and his followers[2] as well as Andrée Ehresmann and Jean-Paul Vanbremeersch [9] and their commentators.[3] But it is still early days, and many approaches need to be tried to find out "where theory lives."

The approach taken here is based on a specific use of the characteristic concepts of category theory, namely universal mapping properties, to define a general schema of determination through universals. The closest approach in the literature (but without the hets and semiadjunctions) is that of François Magnan and Gonzalo Reyes [20] which emphasizes that "Category theory provides means to circumscribe and study what is universal in mathematics and other scientific disciplines." [20, p. 57]. Their intended field of application is cognitive science.

> We may even suggest that universals of the mind may be expressed by means of universal properties in the theory of categories and much of the work done up to now in this area seems to bear out this suggestion....
>
> By discussing the process of counting in some detail, we give evidence that this universal ability of the human mind may be conveniently conceptualized in terms of this theory of universals which is category theory. [20, p. 59][4]

The approach of determination through universals, like any approach, should be judged on how well it isolates and describes the essential and important features of biological and cognitive systems.

2 Adjunctions, hets, and semiadjunctions

In the body of this paper, I will try to keep the mathematics at a minimal conceptual level–which the mathematical formulations restricted to the Appendix. Category theory lends itself to visualization in diagrams so that non-mathematical style of presentation is emphasized by an abundant use of diagrams.

A *category* is intuitively a set of objects of the same type. *Morphisms* between objects should be thought of as a type of determining relation or cause-effect relation between the objects. When a morphism is between objects of the same category, it is called a *homomorphism* or *hom*, and when between objects of different categories it is a *heteromorphism* or *het*.[5] One of the problems in the conventional treatment of category theory[6] is that it tries to ignore heteromorphisms even though hets are a natural part of working mathematics. This leads to certain definitions being rather contrived (to avoid mentioning hets), the usual treatment of adjunctions being the case in point.

Adjunctions will be introduced informally and in the natural manner using hets. The general setting is how the objects in one category (e.g., the "environment" in a life sciences context), the "sending" category, will "affect" or "determine" objects in another category (e.g., "organisms"), the "receiving" category. We start with an object X in the sending category, an object X in the receiving category, and a specific het determination $d : X \dashrightarrow A$ from X to A.[7]

[2] See [27] and that paper's references.

[3] See [17] for Kainen's comments on the Ehresmann-Vanbremeersch approach, Kainen's own approach, and a broad bibliography of relevant papers.

[4] See the example of recursion developed below.

[5] See [11] or [12] for the introduction of heteromorphisms in category theory.

[6] The standard presentation is Mac Lane's text [19] but Lawvere and Schanuel's book [18] is a more conceptual introduction. Magnan and Reyes [20] give an excellent informal treatment of the main universal constructions.

[7] Hets are represented by dashed arrows $\dashrightarrow$ and homs by solid arrows $\rightarrow$.

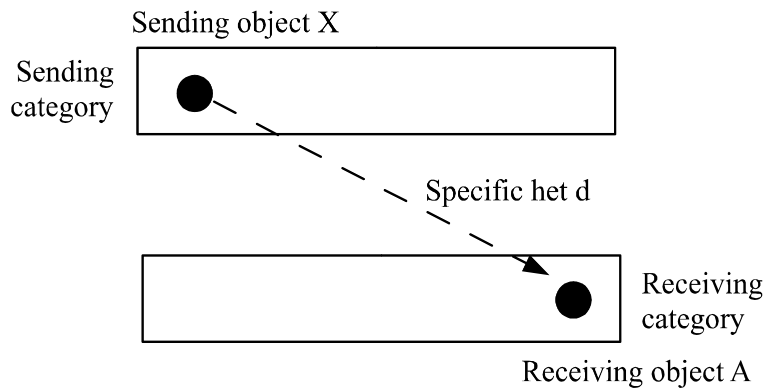

Figure 1: Het $d : X \dashrightarrow A$

One of the most important concepts isolated by category theory (more basic than adjunctions) is the concept of a *universal mapping property* or UMP[8] which models an important type of determination, *determination through universals*. With each sending object X in the sending category, there is an associated *receiving universal* $F(X)$ in the receiving category, and there is a *universal receiving het* $h_X : X \dashrightarrow F(X)$. The universal mapping property is: for every het $d : X \dashrightarrow A$, there is a unique hom $f(d) : F(X) \longrightarrow A$ in the receiving category such that:

$$X \overset{h_X}{\dashrightarrow} F(X) \overset{f(d)}{\longrightarrow} A = X \overset{d}{\dashrightarrow} A$$

i.e., such that the determination through the receiving universal het $h_X : X \dashrightarrow F(X)$ followed by the hom $f(d) : F(X) \to A$ is the same as the original het $d : X \dashrightarrow A$.

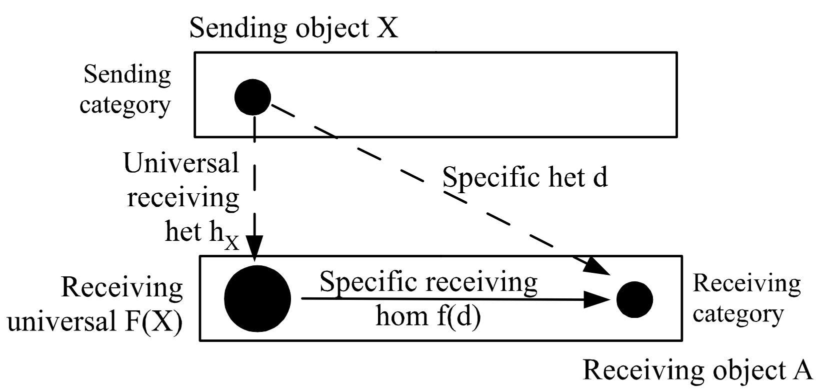

Figure 2: Scheme for determination by a receiving universal $F(X)$.

Moreover given any hom $f : F(X) \to A$ in the receiving category, preceding it by the universal receiving het h_X would give a het $fh_X : X \dashrightarrow A$ which would play the role of $d : X \dashrightarrow A$ in the above equation. Hence there is a one-to-one correspondence (or isomorphism) between the hets $d : X \dashrightarrow A$ and the homs $f : F(X) \to A$. This isomorphism is also canonical or natural (in the category-theoretic sense), and can be represented as:

$$\mathrm{Hom}_{receiving}(F(X), A) \cong \mathrm{Het}(X, A).$$

Even before completing our definition of an adjunction, we can define the above situation, given by the association of the receiving universal $F(X)$ with each object X in the sending category (and

[8]A UMP illustrates the philosophical notion of a concrete universal [10].

similarly for morphisms so it is a "functor") along with the canonical isomorphism $\mathrm{Hom}(F(X), A) \cong \mathrm{Het}(X, A)$, as a *left semiadjunction.* Even if a full adjunction is not present, we will still refer to the functor $F()$ as a *left adjoint* whose values $F(X)$ are the receiving universals.

Then we return to the specific het d and take the dual case which will define the dual notion of a "right semiadjunction." With each receiving object A in the receiving category, there is an associated *sending universal* $G(A)$ in the sending category, and there is a *sending universal het* $e_A : G(A) \dashrightarrow A$. The universal mapping property is: for every het $d : X \dashrightarrow A$, there is a unique hom $g(d) : X \longrightarrow G(A)$ in the sending category such that:

$$X \xrightarrow{g(d)} G(A) \overset{e_A}{\dashrightarrow} A = X \overset{d}{\dashrightarrow} A$$

i.e., such that the determination through the universal sending het $e_A : G(A) \dashrightarrow A$ preceded by the hom $g(d) : X \to G(A)$ is the same as the original het $d : X \dashrightarrow A$.

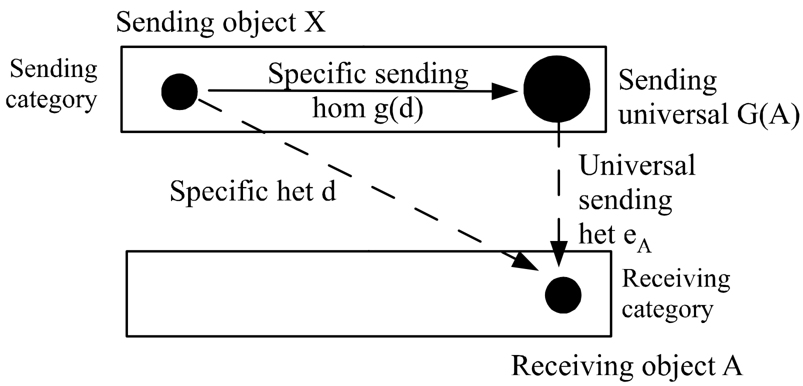

Figure 3: Scheme for determination by a sending universal $G(A)$.

Moreover given any hom $g : X \to G(A)$ in the sending category, following it by the universal sending het e_A would give a het $e_A g : X \dashrightarrow A$ which would play the role of $d : X \dashrightarrow A$ in the above equation. Hence there is a one-to-one correspondence (or isomorphism) between the hets $d : X \dashrightarrow A$ and the homs $g : X \to G(A)$. This isomorphism is also canonical or natural, and can be represented as:

$$\mathrm{Het}(X, A) \cong \mathrm{Hom}_{sending}(X, G(A)).$$

Dually, we can define the above situation, given by the association of the sending universal $G(A)$ with each object A in the receiving category along with the canonical isomorphism $\mathrm{Het}(X, A) \cong \mathrm{Hom}_{sending}(X, G(A))$, as a *right semiadjunction.* Even if a full adjunction is not present, we will still refer to the functor $G()$ as a *right adjoint* whose values are the sending universal $G(A)$.

Now we are prepared to define an *adjunction* essentially as:

$$\text{adjunction} = \text{left semiadjunction} + \text{right semiadjunction}$$
$$\mathrm{Hom}_{receiving}(F(X), A) \cong \mathrm{Het}(X, A) \cong \mathrm{Hom}_{sending}(X, G(A)).$$

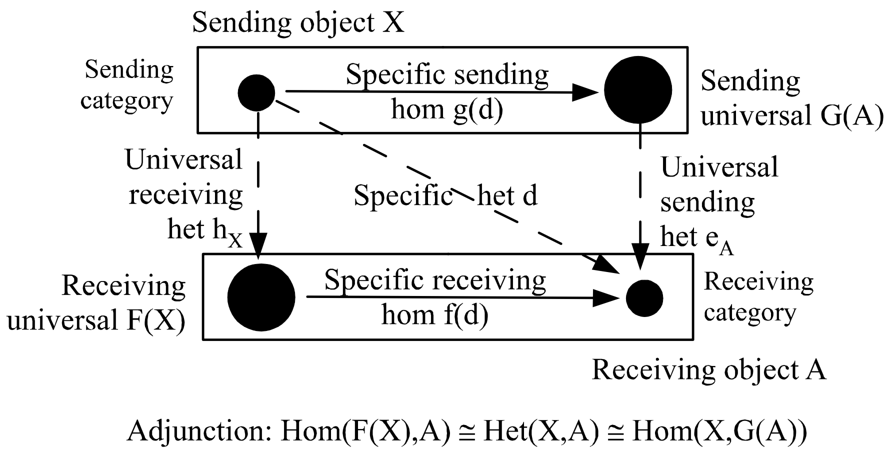

Figure 4: The Adjunctive Square Diagram

Recall that the conventional treatment of an adjunction keeps the hets "in the closet," so the defining natural isomorphism just leaves out the middle het term $\mathrm{Het}(X, A)$ to give the het-free notion of an adjunction: $\mathrm{Hom}(F(X), A) \cong \mathrm{Hom}(X, G(A))$ where $F()$ is the *left adjoint* and $G()$ is the *right adjoint*.

In almost all adjunctions, one of the adjoints is rather trivial and contrived so the principal universal mapping property is expressed by the other adjoint, but one needs the trivial adjoint in order to avoid the hets. This has hampered applications since it is the main receiving or sending universal that is important, not the trivial other adjoint needed to formulate the het-free adjunction. Moreover since determinations between quite different type of objects are important in applications, hets are an essential part of the story, and thus the treatment of adjunction leaving out the hets also hampers applications. But by bringing the hets "out of the closet," we can "fission" the adjunction into two semiadjunctions, one of which is typically appropriate in an application. And we can recombine the two semiadjunctions in a different way to define a "brain functor"–which combines two non-trivial dual semiadjunctions in one scheme of *two-way* determination.

3 Determinations through receiving universals

3.1 Selection versus instruction

In the most generic example of determination through a receiving universal (i.e., the determinative scheme of a left semiadjunction), the sending category or domain is the "environment" while the receiving category or domain is an "organism" (or population of organisms) and the factoring of a het determination $d : X \dashrightarrow A$ through the receiving universal $F(X)$ is a type of selective "recognition." Indeed, the contrast between the direct determination by the het $d : X \dashrightarrow A$ and the factorization through the receiving universal can be seen as the contrast between an instructionist account and a selectionist account of some determination.

We begin with a toy example from Peter Medawar [23, pp. 88-90]. A jukebox contains a "universal" set of records each of which has the instructions for some piece of music. A person in the environment selects and pushes a button which causes the jukebox to supply the musical instructions in the form of a record to the record player part of the mechanism. Alternatively, the person in the environment could directly supply the musical instructions in the form of a record to a record player. In each case, we end up with a record player playing a record but by two different types of mechanisms, one being selective and the other being instructive.

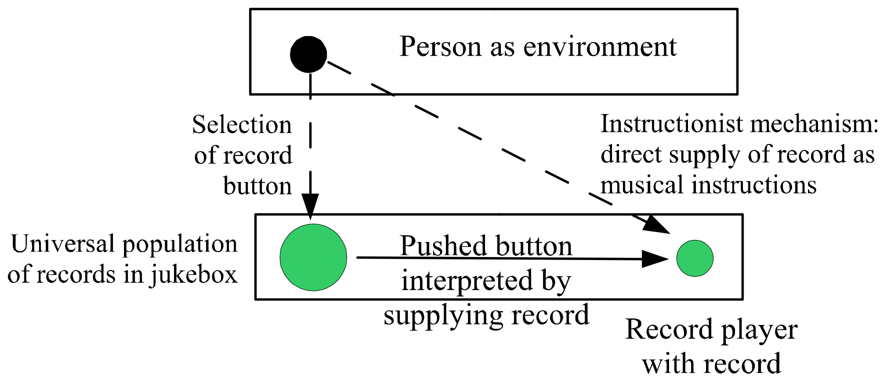

Figure 5: Selection as determination through a receiving universal

3.2 Evolution by natural selection

After Gerald Edelman received the Nobel prize for his work on the selectionist approach to the immune system, he switched to neurophysiology and is thus well-placed to isolate the selectionist commonalities in these different domains (which turn out to be modeled by left semiadjunctions).

> The long trail from antibodies to conscious brain events has reinforced my conviction that evolution, immunology, embryology, and neurobiology are all sciences of recognition whose mechanics follow selectional principles. ...All selectional systems follow three principles. There must be a generator of diversity, a polling process across the diverse repertoires that ensue, and a means of differential amplification of the selected variants.[7, p. 7367]; (also [6, pp.41-2])

These three principles are functionally represented by the three components in a determination through a receiving universal. The "generator of diversity" is the receiving universal object, the "polling process across the diverse repertoires" is represented by the receiving universal morphism that is the canonical external-internal interface between external environment and the receiving universal object, and finally the "differential amplification" is represented by the factor hom.

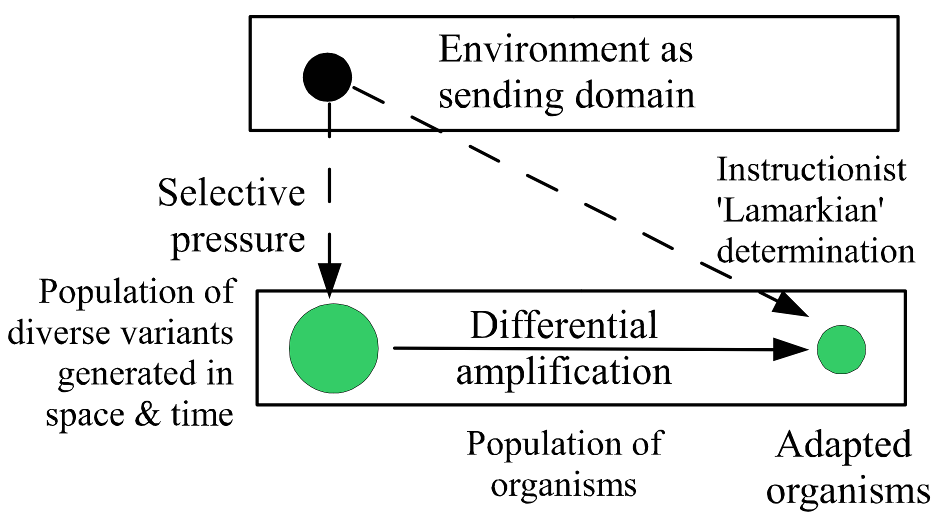

Figure 6: Natural selection as determination through a receiving universal.

The direct determination given by the het would represent a Lamarckian instructionist determination; the environment directly inducing the changes in the organism to adapt it to the environment.

The indirect determination through the receiving universal using the "universal" variations in organisms as the basis for the environment's Darwinian natural selection so that the adapted organisms differentially reproduce.

3.3 Immune system as a selectionist mechanism

Evolution is not the only example in the life sciences of determinative processes that were originally assumed to be instructionist but were later found to operate by a selectionist mechanism. One of the most telling cases was the immune system. Originally it was assumed that the antigen would somehow instruct the immune system as to how an anti-body could be constructed to neutralize the antigen. During the 1950s, a number of difficulties in the instructionist account fostered the development of a selectionist approach. While many researchers contributed to this approach, one of the earliest was Niels Jerne [15] who has also been most attentive to analogies with other fields.

In the selectionist theory, the immune system takes on the active role of generating a huge well-nigh "universal" variety of antibodies but in low concentrations. This initial generation of candidate antibodies is not being directed or instructed by the past disease history of the organism. An externally introduced antigen has the indirect role of simply selecting which antibody fits it like a key in a lock. Every antibody has the possibility of self-reproducing or cloning itself but it is the ones whose key has fit into a lock that have this potentiality triggered. Then that antibody is differentially amplified in the sense of being cloned into many copies to lock up the other instances of the antigen. Thus the selectionist account of the immune system has the main features of determination through a receiving universal.

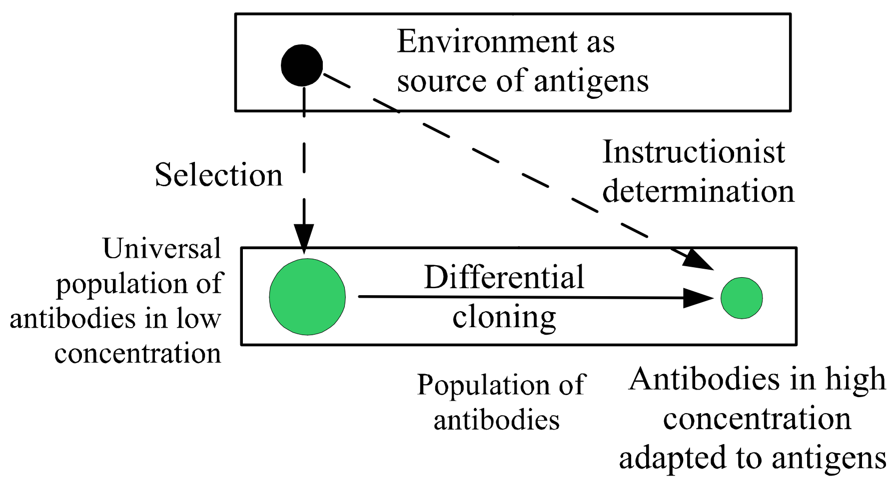

Figure 7: Immune system as determination through a receiving universal.

3.4 Edelman's neural Darwinism

Edelman carried over the selectionist model to neurophysiology to develop his theory of neuronal group selection or neural Darwinism.

> [T]he theoretical principle I shall elaborate here is that the origin of categories in higher brain function is somatic selection among huge numbers of variants of neural circuits contained in networks created epigenetically in each individual during its development; this selection results in differential amplification of populations of synapses in the selected variants. In other words, I shall take the view that the brain is a selective system more akin in its workings to evolution than to computation or information processing.[5, p. 25]

There are several different phases in this selectionist theory. In the developmental phase of the brain, a huge variety of loose connections are made. Those that find some resonance with the individual's experience are strengthen while those that are unused will atrophy. The slogan is that "the neurons that fire together, wire together." Later there is an experiential selection the strengthens some connections and weakens others. Finally, "reentrant" signals within the brain deepen the process of self-organization through strengthening some connections and weakening others.

In broad-brush terms, one might intuitively think of the universal model as a large set of brain circuits representing a wide ("universal") range of sensory images and vibrating at a low level of amplitude beneath the level of consciousness (analogous to the "universal" repertoire of antibodies present in the immune system in low concentrations). When a specific signal is received from the environment, then it might resonate with a particular circuit-image which would greatly increase the amplitude of those vibrations and would thus constitute the perception. This sort of model has a type of intentionality (i.e., seeing is always seeing-as) since the perception would always be "perception-as" depending on which image was resonated.

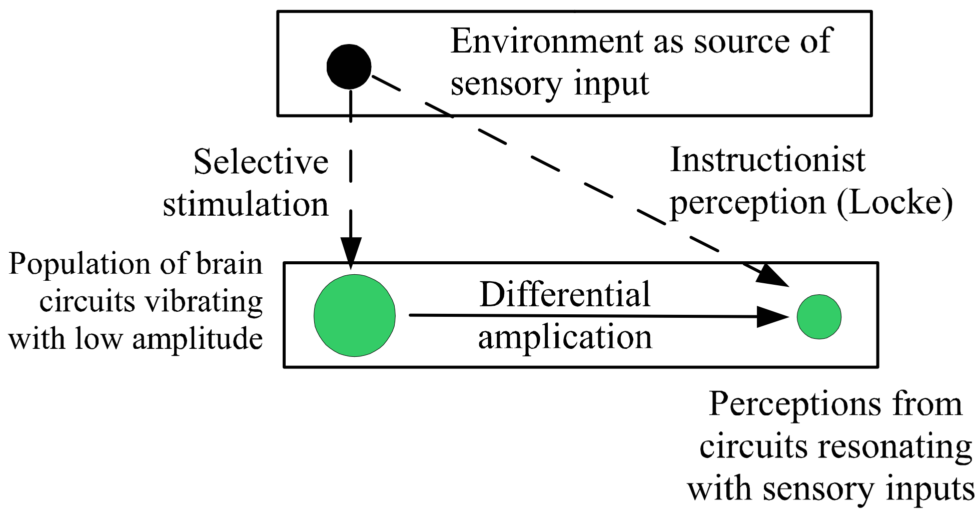

Figure 8: Edelman's perception as determination through a receiving universal.

In Edelman's account of perception as the "remembered present," direct determination is distinguished from the composite effect of the indirect influence differentially triggering internal processes.

> According to this analysis, extrinsic signals convey information not so much in themselves, but by virtue of how they modulate the intrinsic signals exchanged within a previously experienced neural system. In other words, a stimulus acts not so much by adding large amounts of extrinsic information that need to be processed as it does by amplifying the intrinsic information resulting from neural interactions selected and stabilized by memory through previous encounters with the environment. [8, p. 137]

3.5 Active versus passive learning

This recalls a much older theme of "recollection." One of the tell-tale signs of a process of determination through universals is the indirectness of the factorization through a universal. Here again, an instructionist account might be first given for a process that is later recognized as being selectionist. The interplay between these two accounts dates back at least to the Platonic-Socratic account of learning not as the result of external instruction but as a process of catalyzing internal recollection. One of the striking epigrams of neo-Platonism is the thesis that "no man ever does or can teach another anything" [2, p. 1]. In the early fifth century, Augustine in *De Magistro* (The Teacher) made the point contrasting "outward" passive instruction with active learning "within."

> But men are mistaken, so that they call those teachers who are not, merely because for the most part there is no delay between the time of speaking and the time of cognition. And since after the speaker has reminded them, the pupils quickly learn within, they think that they have been taught outwardly by him who prompts them.(Chapter XIV)

In the nineteenth century, Wilheim von Humboldt made the same point even recognizing the symmetry between listener and speaker (a symmetry captured by the right semiadjunctions).

> Nothing can be present in the mind (Seele) that has not originated from one's own activity. Moreover understanding and speaking are but different effects of the selfsame power of speech. Speaking is never comparable to the transmission of mere matter (Stoff). In the person comprehending as well as in the speaker, the subject matter must be developed by the individual's own innate power. What the listener receives is merely the harmonious vocal stimulus. [14, p. 102]

A similar theme has been a mainstay in active learning theories of education. As John Dewey put it:

> It is that no thought, no idea, can possibly be conveyed as an idea from one person to another. When it is told, it is, to the one to whom it is told, another given fact, not an idea. The communication may stimulate the other person to realize the question for himself and to think out a like idea, or it may smother his intellectual interest and suppress his dawning effort at thought. [4, p. 159]

Remarkably, the immunologist Niels Jerne tied these themes together.

> Several philosophers, of course, have already addressed themselves to this point. John Locke held that the brain was to be likened to white paper, void of all characters, on which experience paints with almost endless variety. This represents an instructive theory of learning, equivalent to considering the cells of the immune system void of all characters, upon which antigens paint with almost endless variety.
>
> Contrary to this, the Greek Sophists, including Socrates, held a selective theory of learning. Learning, they said, is clearly impossible. For either a certain idea is already present in the brain, and then we have no need of learning it, or the idea is not already present in the brain, and then we cannot learn it either, for even if it should happen to enter from outside, we could not recognize it. This argument is clearly analogous to the argument for a selective mechanism for antibody formation, in that the immune system could not recognize the antigen if the antibody were not already present. Socrates concluded that all learning consists of being reminded of what is pre-existing in the brain.[16, pp. 204-5]

3.6 Chomsky's language acquisition faculty

Niels Jerne also saw the connection with another major example of a left semiadjunction, Norm Chomsky's theory of language acquisition. Language learning by a child is another example of a process that was originally thought to be instructive. But Noam Chomsky's theory of generative grammar postulated an innate language faculty that would unfold according to the linguistic experience of the child. The child did not "learn" the rules of grammar; the linguistic experience of the child would select how the universal language faculty would develop or unfold to differentially implement one rule rather than another. Niels Jerne entitled his Nobel Lecture, *The Generative Grammar of the Immune System.*

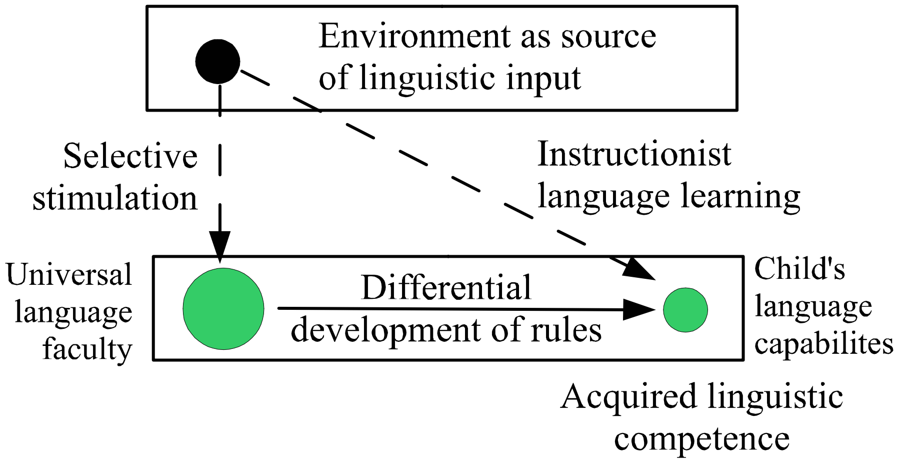

Figure 9: Language acquisition as determination through a receiving universal.

Chomsky [3] emphasizes certain aspects of the language acquisition faculty which can be illustrated as general aspects of left semiadjunctions:

- *Rich profligate internal structure*: the capacity to generate 'all' the various possibilities with the receiving universal;
- *Innateness*: that receiving universal is on the receiving ("organism") side, not the sending ("environment") side, even though it "recognizes" the inputs from the sending side;
- *Impoverished or minimal inputs*: for instance, selecting the jukebox button inputs less information that the whole instructive message of the supplied record in Medawar's example;
- *Active role of internal mechanism*: the selection interacts with the universal receiver to generate a specific internal determination rather than passively receiving the detailed instructionist message like a stamp in wax;
- *Relative autonomy of internal mechanism*: the active internal role together with the impoverished external input (and lack of stimulus-control) gives the determination through the receiving universal a certain type of autonomy from the sending environment.

Since one of the principal capabilities of Chomsky's language faculty is recursion, we might make a slight mathematical digression to notice that the mathematical formulation of recursion is a left semiadjunction.

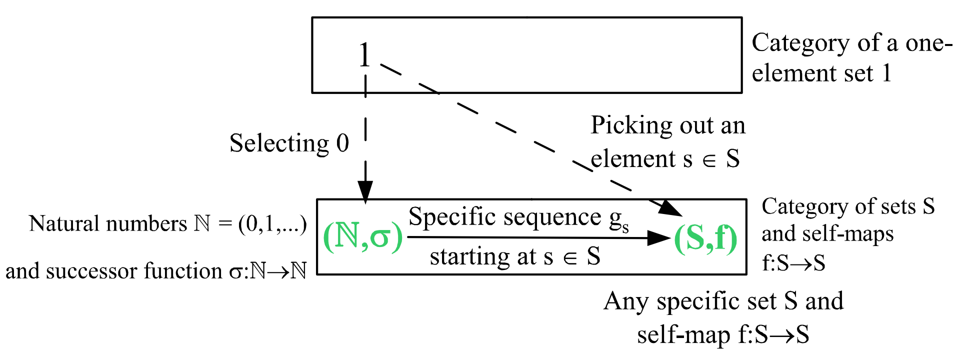

Figure 10: Recursion as determination through a receiving universal.

The specific sequence g_s enumerated by the natural numbers is $s, f(s), f^2(s), f^3(s), \ldots$.

3.7 Language understanding

Not only the acquisition of language but the ordinary understanding of language is another example well-modeled by determination through a receiving universal. If the auditory input is in a language that the listener understands, that means there is an internal process triggered by the auditory signals that recognizes, interprets, and understands the input. The Lockean or behaviorist alternative is where the auditory input (het), Humboldt's "vocal stimulus," is directly supplied by the auditory version of Locke's writing on a blank slate.

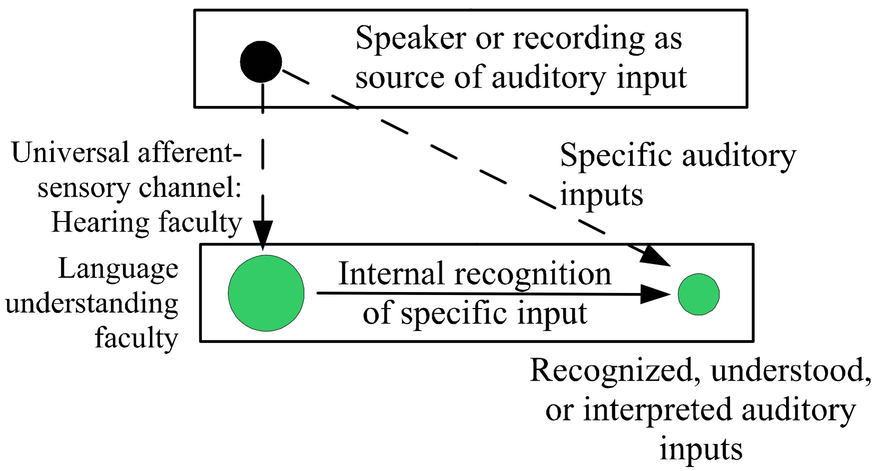

Figure 11: Language understanding through a receiving universal.

The determination through the receiving universal thus adds a second level to the input which is variously called recognition, understanding, or the *intentionality of perception*. This second level is often indicated by saying that the sensory input is "perceived as", "recognized as", or "understood as" something further.

3.8 Generic "recognition" or "perception"

Before turning to right semiadjunctions, it might be useful to present a rather generic version of determination through a receiving universal as model of "recognition" or "perception" that captures many of the common features of the various examples.

The determination through the receiving universal is the active internal process that supplies the "interpretation" or "intentionality" to the raw sense data. The red blotch is seen as a tomato; the sound "ya" is understood as indicating agreement, and so forth. In the direct alternative, the raw sensory input supplies Lockean "perception" like writing on a blank slate or a stamp making an impression on wax.

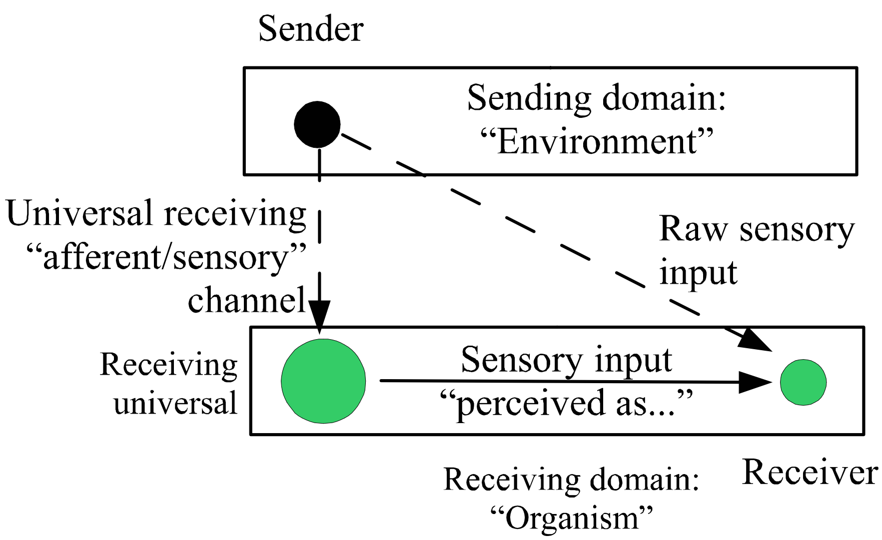

Figure 12: "Recognition" as determination through a receiving universal.

4 Determinations through a sending universal

4.1 Generic "action"

Dual to the generic model of "recognition" is the generic model of "action"–which is the determinative scheme given by a right semiadjunction. In the model of recognition, there is the uninterpreted message as just a sensory input (the external het), and then there is the second level where the factorization (the internal hom) through the receiving universal recognizes the interpretation, meaning, or intentionality of the message. In the dual model of "action," the external het specifies the external behavior (which could be even a reflex behavior) while internal hom factoring through a sending universal that supplies the "intentionality" of the "action" (where an "action" is a "behavior" with the second level of "intentionality"). In each case, we end up with a certain behavior but determined by two different means.

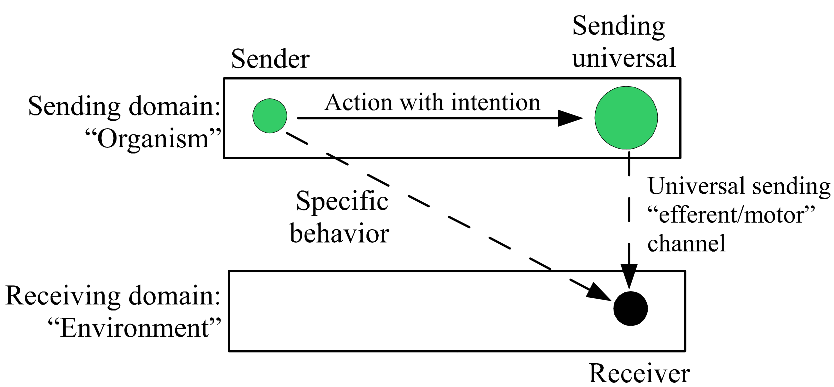

Figure 13: "Action" as determination through a sending universal.

4.2 Universal Turing machines

Universal Turing machines provide a good example of a sending universal and they are strikingly similar to the next example of the genetic code. Given the inputs, a specific Turing machine (TM) might calculate a function such as the successor function. But if those inputs and the coding for that specific TM were fed in as inputs to a universal TM, then it would realize the same calculated results.

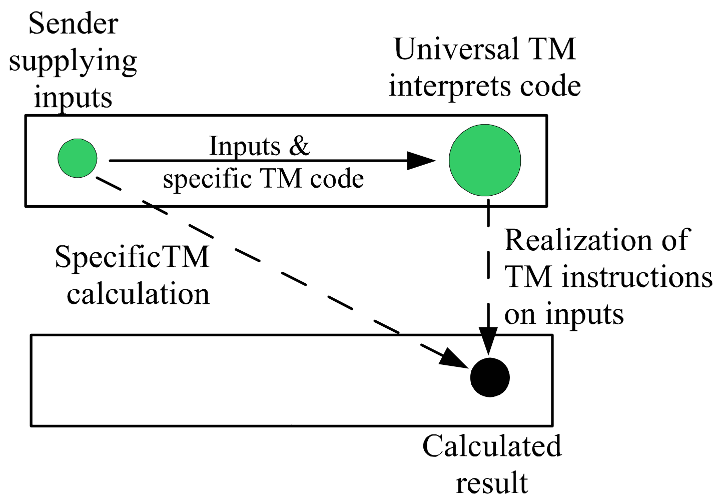

Figure 14: Universal Turing machine as a sending universal.

4.3 The genetic code and DNA mechanism

Surely the most important example of a right semiadjunction in biology is the whole DNA mechanism to use the genetic code to construct amino acids. One might imagine a specific chemical mechanism that would produce a specific amino acid–which would play the role of the specific sending het. But the DNA mechanism uses an internal sending universal to interpret coded messages to make one amino acid or another, and then to realize the construction of that amino acid.

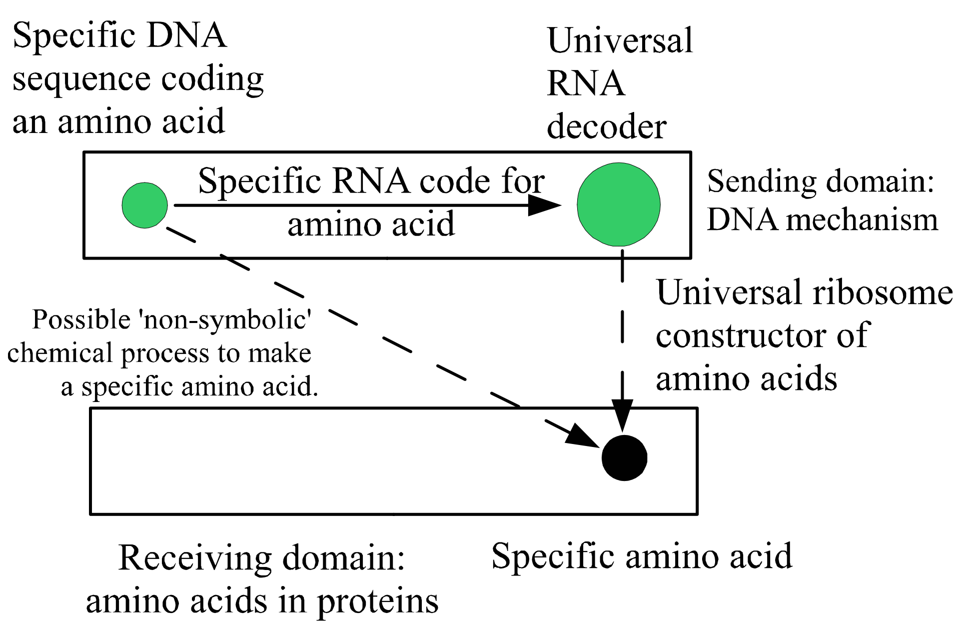

Figure 15: DNA mechanism as a determination through a sending universal.

John Maynard Smith has emphasized that "the use of informational terms implies intentionality" [21, p. 124] which is that second level, the chemistry plus the coded meaning of the chemistry. The "meaning" is supplied not by a human sender but by the process of evolution which essentially sends the message: "this form has survival value."

Furthermore it is the use of coded information that allows for the generation of such "universal" variety for selection to operate upon:

> I think that it is the symbolic nature of molecular biology that makes possible an indefinitely large number of biological forms. [21, p. 133]

> The mere existence of replicators is not sufficient for continued evolution by natural selection, which requires what we have called "indefinite hereditary replicators": that is, entities that can exist in an indefinitely large number of states, each of which can be replicated. In living organisms, nucleic acid molecules are the only indefinite hereditary replicators, or at least they were until the invention of language and music. [22, p. 58]

4.4 Developmental mechanisms

Less well understood is the way that informational codes drive hierarchies of regulator genes to develop this or that organ. But the whole arrangement still has the form of a right semiadjunction. The contrast between a specific het and the corresponding specific hom (with its second level of meaning or intentionality) is explained by Maynard Smith using the example of proteins.

> A protein might have a function directly determined by its structure–for example, it may be a specific enzyme, or a contractile fiber. Alternatively, it might have a regulatory function, switching other genes on or off. Such regulatory functions are arbitrary, or symbolic. They depend on specific receptor DNA sequences, which have themselves evolved by natural selection. The activity of an enzyme depends on the laws of chemistry and on the chemical environment (for example, the presence of a suitable substrate), but there is no structure that can be thought of as an evolved "receiver" of a "message" from the enzyme. By contrast, the effect of a regulatory protein does depend on an evolved receiver of the information it carries: the eyeless gene signals "make an eye here," but only because the genes concerned with making an eye have an appropriate receptor sequence. [21, p. 143]

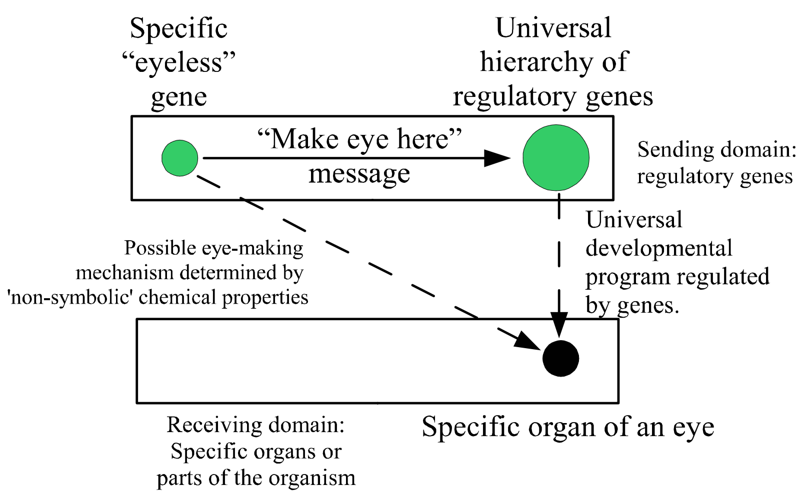

Figure 16: Hierarchy of regulatory genes as a sending universal.

A stem cell would fit into the same type of diagram as the sending universal. One could imagine some specific non-symbolic chemical process (playing the role of the specific het) that would develop a certain type of bodily cell. But a stem cell is a sending universal in that upon receiving the proper specific chemical codes (playing the role of the corresponding specific hom), it will develop into some specific type of cell.

4.5 Language action

The dual to "language understanding" is language production or linguistic action (e.g., "speech acts"). The role of the specific het is played by some auditory output such as utterances (Humboldt's "vocal stimulus"). But the corresponding internal specific hom is the speech act (i.e., internal speech with intentionality) that through the language faculty produces the same outputs but as intentional speech.

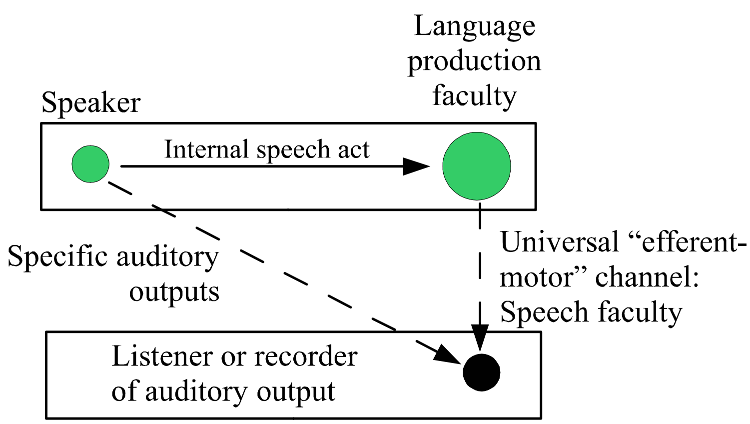

Figure 17: Language production through a sending universal.

5 Brain Functors

5.1 Recombination of left and right semiadjunctions

It might be noted that in a number of our examples essentially the same faculty, e.g., the language faculty, plays both the role of the receiving or recognition universal and the sending or action universal. Hence it should be possible to recombine the semiadjunctions so that the two universals coincide, and that yields the concept of the brain functor (the "brain" being that universal for both recognition and action). From the mathematical viewpoint, the key to this was using the hets to split an adjunction into left and right semiadjunctions which can then be recombined in the opposite way.

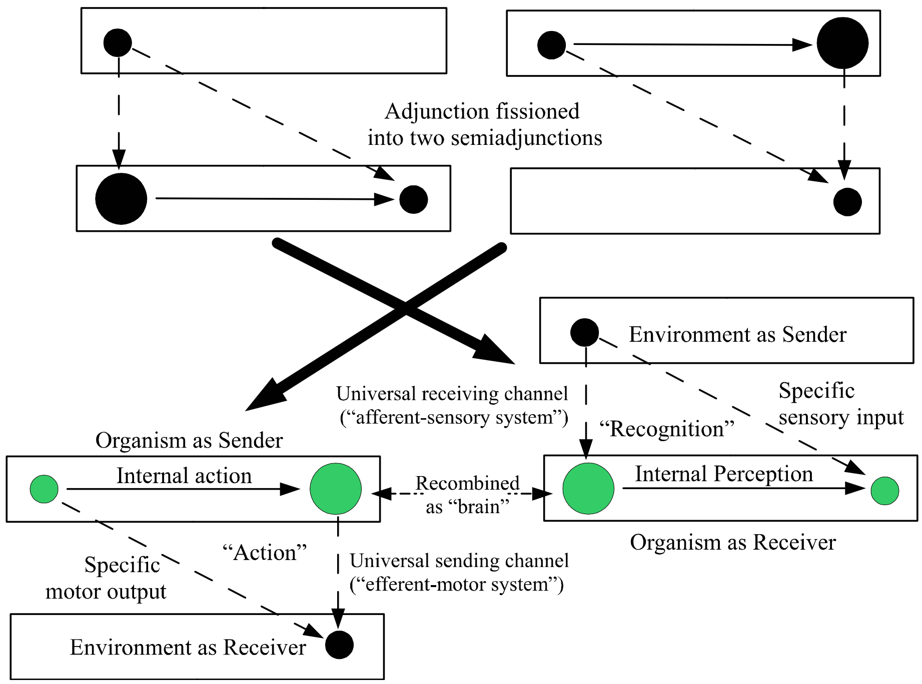

Figure 18: Recombining a fissioned adjunction to make a brain functor.

The recombined semiadjunctions then form a *brain functor*. In the following ‘butterfly’ for a brain functor, we use labels appropriate to the "brain" label.

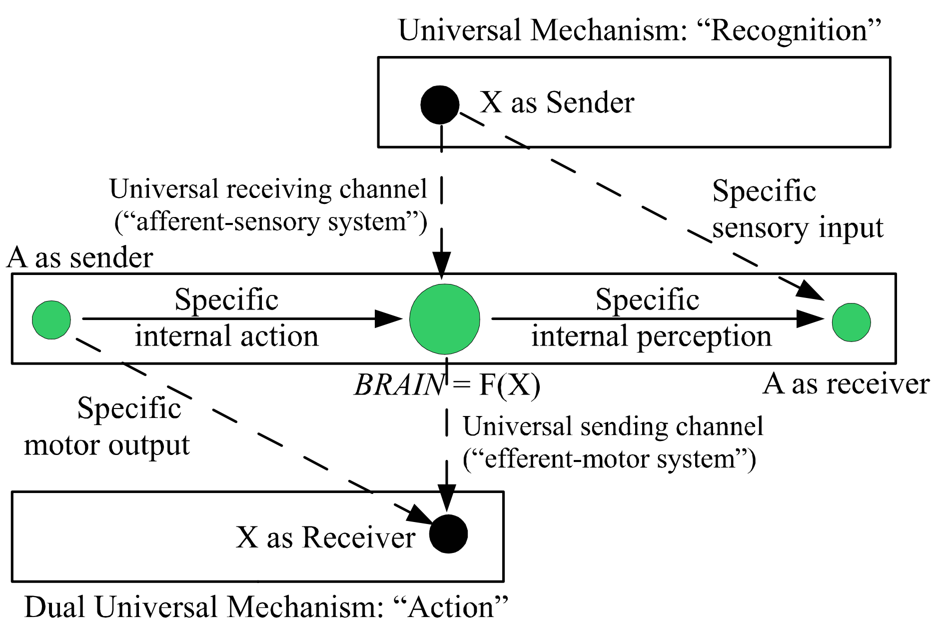

Figure 19: Brain: Scheme for receiving and sending through one universal.

Mathematically, for each het $X \dashrightarrow A$ there is a unique internal "recognizing" hom $F(X) \rightarrow A$ so there is a canonical isomorphism: $Hom(F(X), A) \cong Het(X, A)$. And for each het $A \dashrightarrow X$ in the other direction, there is a unique internal "action" hom $A \rightarrow F(X)$ so there is also a canonical isomorphism: $Het(A, X) \cong Hom(A, F(X))$.

The concept of a brain functor is the natural cognate or associated concept to an adjunction. For an adjunction, there are *two* functors that represent on the left and right the hets going *one* way between the categories:

$$Hom(F(X), A) \cong Het(X, A) \cong Hom(X, G(A)).$$

For a brain functor, there is *one* functor that represents on the left and right the hets going the *two* ways between the categories:

$$Hom(F(X), A) \cong Het(X, A) \text{ and } Het(A, X) \cong Hom(A, F(X)).$$

Hence the general scheme given by a brain functor is *receiving and sending determination through one universal* (the "brain").

5.2 Simple two-way determinations through one universal

The simplest form of a brain "functor" is just a two-way representation or coding system that constructs and implements a set of codes. Given some set of objects, it is encoded using some isomorphic set of representations or codes for the objects, and then given an instance of the code, it is decoded to determine the object.

Coordinatizing is a form of coding. The geometrical plane is a collection of points, and the Cartesian coordinate system represents each point P by a pair (x_P, y_P) of coordinates. Given a point P, the "coordinate" function selects the coordinates (x_P, y_P) of the point which is the recognized or coded output, and given the coordinates or code for a point (x_P, y_P) as an input, the "plot" function designates the point.

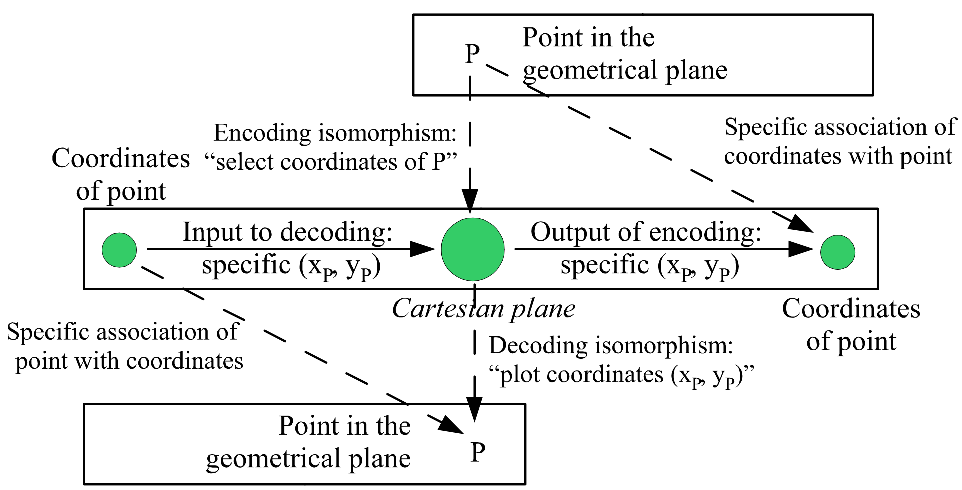

Figure 20: Coding and decoding Cartesian coordinates of geometrical points.

5.3 The language faculty

A brain functor, broadly put, is any universal mechanism of determination that can factor determination either way through a universal–rather than an adjunction that factors one way determination through two (receiving and sending) universals. In some contexts in the life sciences, determination is strictly one way so one might expect to find a semiadjunction but not a two-way system like a brain functor. For instance, there is the *fundamental dogma* that DNA determines amino acids, proteins, and eventually the characteristics of an organism but never the reverse.

An application of the scheme for a brain functor in the cognitive sciences is to model the language faculty where there is two way determination between vocal stimuli and internal representations.

The previous semiadjunctions for language understanding and language action can be merged to arrive at the brain-like function of the language faculty.

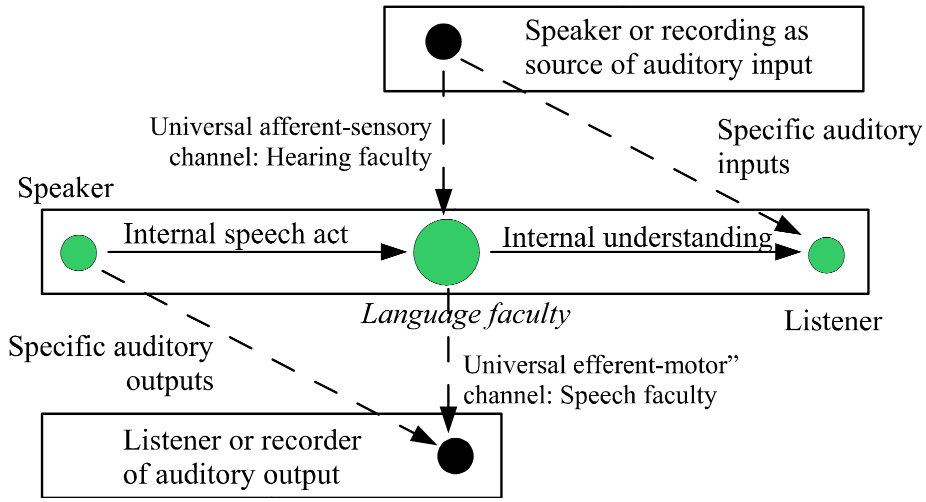

Figure 21: Language faculty as two-way determination through a universal.

6 Summary

The following table gives the principal exact category-theoretic concepts to describe the corresponding schemes of determination through universals as well as the main generic examples.

CT concept	**Determination through universals**	**Generic example**
Left semiadjunction	Det. through a receiving universal	Recognition
Right semiadjunction	Det. through a sending universal	Action
Brain functor	Two-way det. through a universal	Recognition + Action

Table 1: Principal forms of determination through universals

The importance of category-theoretical universals in isolating the important concepts in pure mathematics suggests that the universals may play a similar role in the empirical sciences. Our results suggest that this is indeed the case in the biological and cognitive sciences. The category-theoretic schemes of determination through universals are at a high level of abstraction, but, in this case at least, that seems to be where some significant theory lives. Regardless of the great differences in the underlying substrate processes, many of the most important mechanisms and faculties of the biological and cognitive sciences not only seem to fit into, but also to have their key features characterized by, the schemes for determination through universals.

7 Appendix: Defining hets in category theory

Category theory groups together in *categories* the mathematical objects with some common structure (e.g., sets, partially ordered sets, groups, rings, and so forth) and the appropriate morphisms between such objects. Since the morphisms are between objects of similar structure, they are ordinarily called "homomorphisms."

But there have always been other morphisms which occur in mathematical practice that are between objects with different structures (i.e., in different categories) such as the insertion-of-generators map from a set to the free group on that set. Indeed, the working mathematician might well characterize the free group $F(X)$ on a set X as the group such that for any set-to-group map $f : X \dashrightarrow G$, there is a unique group homomorphism $\widehat{f} : F(X) \to G$ that factors f through the canonical insertion of generators $i : X \dashrightarrow F(X)$, i.e., $f = \widehat{f}i$. In order to contrast these morphisms such as $f : X \dashrightarrow G$ and $i : X \dashrightarrow F(X)$ with the homomorphisms between objects within a category such as $\widehat{f} : F(X) \to G$, the former are called *heteromorphisms* or *hets* (for short). Hets are like chimeras since they have a tail in one category and a head in another category.

We assume familiarity with the usual machinery of category theory (bifunctors, in particular) which can be adapted to give a rigorous treatment of heteromorphisms (and their compositions with homomorphisms) that is parallel to the usual bifunctorial treatment of homomorphisms.

The cross-category object-to-object hets $d : X \dashrightarrow A$ will be indicated by dashed arrows ($\dashrightarrow$) rather than solid arrows ($\to$). The first question is how do heteromorphisms compose with one another? But that is not necessary. Chimera do not need to 'mate' with other chimera to form a 'species' or category; they only need to mate with the intra-category morphisms on each side to form other chimera.[9]

Given a het $d : X \dashrightarrow A$ from an object in a category $\mathbf{X}$ to an object in a category $\mathbf{A}$, and homs $h : X' \to X$ in $\mathbf{X}$ and $k : A \to A'$ in $\mathbf{A}$, the composition $dh : X' \to X \dashrightarrow A$ is another het $X' \dashrightarrow A$ and the composition $kd : X \dashrightarrow A \to A'$ is another het $X \dashrightarrow A'$.

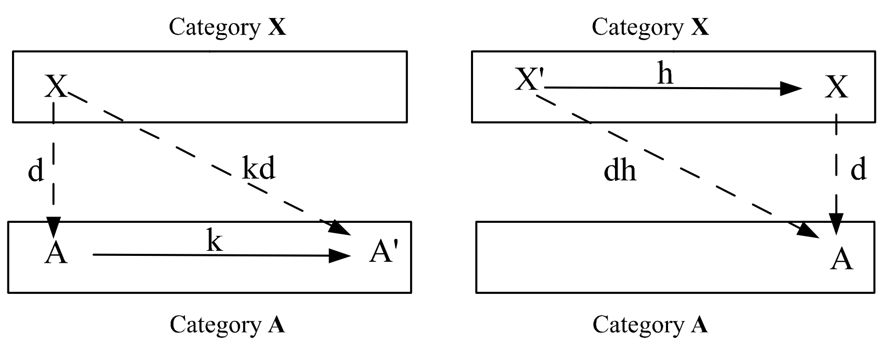

Figure 22: Composition of hets and homs.

This action is exactly described by a bifunctor $\mathrm{Het} : \mathbf{X}^{op} \times \mathbf{A} \to \mathbf{Set}$ where $\mathrm{Het}(X, A) = \{X \dashrightarrow A\}$ and where $\mathbf{Set}$ is the category of sets and set functions. The natural machinery to treat object-to-object morphisms *between* categories are het-bifunctors $\mathrm{Het} : \mathbf{X}^{op} \times \mathbf{A} \to \mathbf{Set}$ that generalize the hom-bifunctors $\mathrm{Hom} : \mathbf{X}^{op} \times \mathbf{X} \to \mathbf{Set}$ used to treat object-to-object morphisms *within* a category.

For any $\mathbf{A}$-hom $k : A \to A'$ and any het $X \overset{d}{\dashrightarrow} A$ in $\mathrm{Het}(X, A)$, there is a composite het $X \overset{d}{\dashrightarrow} A \xrightarrow{k} A' = X \overset{kd}{\dashrightarrow} A'$, i.e., k induces a map $\mathrm{Het}(X, k) : \mathrm{Het}(X, A) \to \mathrm{Het}(X, A')$. For any $\mathbf{X}$-hom $h : X' \to X$ and het $X \overset{d}{\dashrightarrow} A$ in $\mathrm{Het}(X, A)$, there is the composite het $X' \xrightarrow{h} X \overset{d}{\dashrightarrow} A = X' \overset{dh}{\dashrightarrow} A$, i.e., h induces a map $\mathrm{Het}(h, A) : \mathrm{Het}(X, A) \to \mathrm{Het}(X', A)$ (note the reversal of direction). The induced maps would respect identity and composite morphisms in each category. Moreover, composition is associative in the sense that $(kd)h = k(dh)$. This means that the assignments of sets of chimera morphisms $\mathrm{Het}(X, A) = \{X \overset{d}{\dashrightarrow} A\}$ and the induced maps between them constitute a *bifunctor* $\mathrm{Het} : \mathbf{X}^{op} \times \mathbf{A} \to \mathbf{Set}$ (contravariant in the first variable and covariant in the second).

With this motivation, we may turn around and define *heteromorphisms* from $\mathbf{X}$-objects to $\mathbf{A}$-objects as the elements in the values of a given bifunctor $\mathrm{Het} : \mathbf{X}^{op} \times \mathbf{A} \to \mathbf{Set}$. This would be analogous to defining the homomorphisms in $\mathbf{X}$ as the elements in the values of a given hom-bifunctor $\mathrm{Hom}_{\mathbf{X}} : \mathbf{X}^{op} \times \mathbf{X} \to \mathbf{Set}$ and similarly for $\mathrm{Hom}_{\mathbf{A}} : \mathbf{A}^{op} \times \mathbf{A} \to \mathbf{Set}$.

Given any bifunctor $\mathrm{Het} : \mathbf{X}^{op} \times \mathbf{A} \to \mathbf{Set}$, it is *representable on the left* if for each $\mathbf{X}$-object X, there is an $\mathbf{A}$-object $F(X)$ that represents the functor $\mathrm{Het}(X, -)$, i.e., there is an isomorphism

[9] The chimera genes are dominant in these mongrel matings. While mules cannot mate with mules, it is 'as if' mules could mate with either horses or donkeys to produce other mules.

$\psi_{X,A} : \mathrm{Hom}_{\mathbf{A}}(F(X), A) \cong \mathrm{Het}(X, A)$ natural in A. This defines a functor $F : \mathbf{X} \to \mathbf{A}$, and any such functor with natural isomorphisms $\mathrm{Hom}_{\mathbf{A}}(F(X), A) \cong \mathrm{Het}(X, A)$ is a *left semiadjunction.*

Given a bifunctor Het : $\mathbf{X}^{op} \times \mathbf{A} \to \mathbf{Set}$, it is *representable on the right* if for each $\mathbf{A}$-object A, there is an $\mathbf{X}$-object $G(A)$ that represents the functor $\mathrm{Het}(-, A)$, i.e., there is an isomorphism $\varphi_{X,A} : \mathrm{Het}(X, A) \cong \mathrm{Hom}_{\mathbf{X}}(X, G(A))$ natural in X. This defines a functor $G : \mathbf{A} \to \mathbf{X}$, and any such functor with natural isomorphisms $\mathrm{Het}(X, A) \cong \mathrm{Hom}_{\mathbf{X}}(X, G(A))$ is a *right semiadjunction.*

An *adjunction* is given by two functors $F : \mathbf{X} \to \mathbf{A}$ and $G : \mathbf{A} \to \mathbf{X}$ that form left and right semiadjunctions, i.e.,

$$\mathrm{Hom}_{\mathbf{A}}(F(X), A) \cong \mathrm{Het}(X, A) \cong \mathrm{Hom}_{\mathbf{X}}(X, G(A))$$

natural for all X, A. The identity hom $1_{F(X)} \in \mathrm{Hom}_{\mathbf{A}}(F(X), F(X))$ is associated by the left isomorphism with the *universal receiving het* $h_X : X \dashrightarrow F(X)$, and the identity hom $1_{G(A)} \in \mathrm{Hom}_{\mathbf{X}}(G(A), G(A))$ is associated by the right isomorphism with the *universal sending het* $e_A : G(A) \dashrightarrow A$. Then for any het $d \in \mathrm{Het}(X, A)$, the uniquely associated hom $f(d) \in \mathrm{Hom}_{\mathbf{A}}(F(X), A)$ gives: $d = f(d)\, h_X$. And for the same d, the uniquely associated $g(d) \in \mathrm{Hom}_{\mathbf{X}}(X, G(A))$ gives: $d = e_A g(d)$. Combining the two gives the adjunctive square diagram used previously in the text.

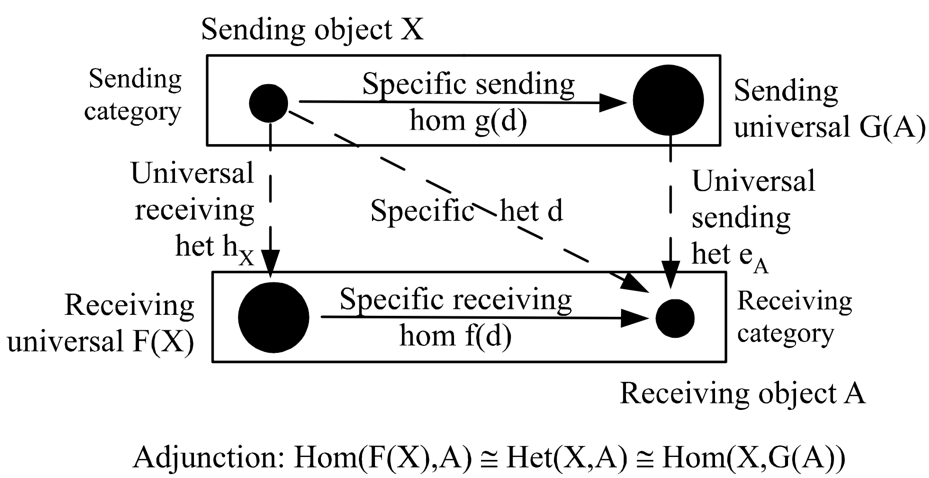

Figure 23: Adjunctive square diagram.

Finally, a *brain functor* is a functor $F : \mathbf{X} \to \mathbf{A}$ that is a left semiadjunction for $\mathrm{Het}(X, A)$ and a right semiadjunction for $\mathrm{Het}(A, X)$, i.e.,

$$\mathrm{Hom}_{\mathbf{A}}(F(X), A) \cong \mathrm{Het}(X, A) \text{ and } \mathrm{Het}(A, X) \cong \mathrm{Hom}_{A}(A, F(X)).$$

For each $d \in \mathrm{Het}\,(X, A)$, there is a unique hom $f\,(d) \in \mathrm{Hom}_{\mathbf{A}}(F(X), A)$ so that the upper triangular 'wing' in the butterfly diagram commutes. For each $d' \in \mathrm{Het}\,(A, X)$, there is a unique hom $g(d') \in \mathrm{Hom}_{A}(A, F(X))$ so that the lower triangular 'wing' commutes.

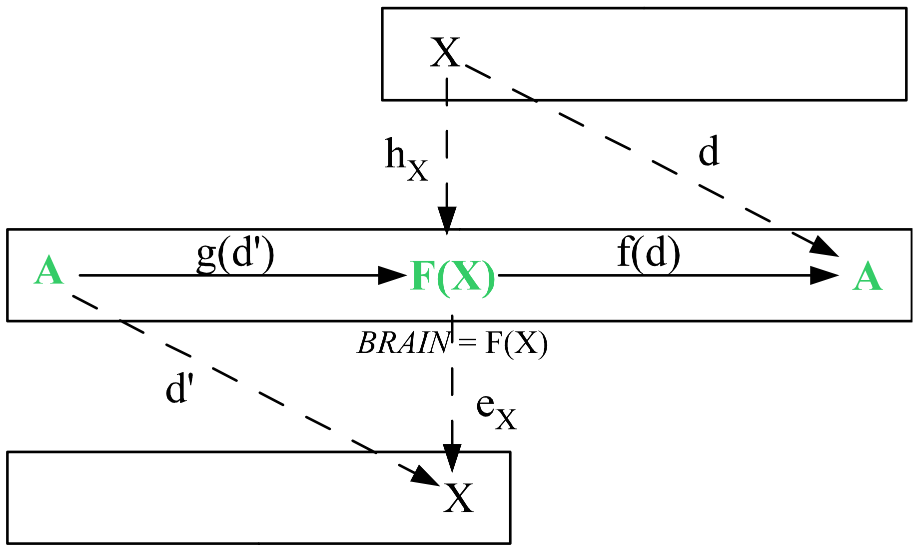

Figure 24: Mathematical butterfly diagram for a brain functor.

If a functor $F : \mathbf{X} \to \mathbf{A}$ has a right adjoint $G : \mathbf{A} \to \mathbf{X}$, then:

$$\mathrm{Hom}_{\mathbf{A}}(F(X), A) \cong \mathrm{Het}(X, A) \cong \mathrm{Hom}_{\mathbf{X}}(X, G(A)).$$

If the functor F also has a left adjoint $H : \mathbf{A} \to \mathbf{X}$, then:

$$\mathrm{Hom}_{\mathbf{X}}(H(A), X) \cong \mathrm{Het}\,(A, X) \cong \mathrm{Hom}_{\mathbf{A}}(A, F(X)).$$

Then taking the isomorphisms that do not involve G or H gives $\mathrm{Hom}_{\mathbf{A}}(F(X), A) \cong \mathrm{Het}(X, A)$ and $\mathrm{Het}\,(A, X) \cong \mathrm{Hom}_{\mathbf{A}}(A, F(X))$, i.e., F is a brain functor. Hence all functors F that have both right and left adjoints are brain functors.

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